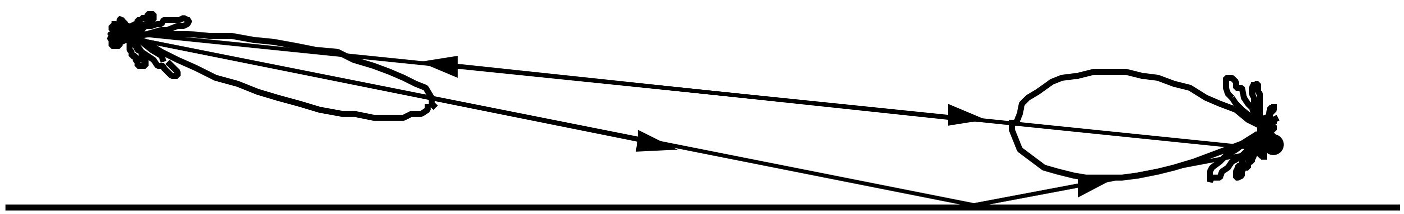
Naval Postgraduate School *Department of Electrical & Computer Engineering* *Monterey, California*

EC3630 Radiowave Propagation

PROPAGATION NEAR THE EARTH'S SURFACE

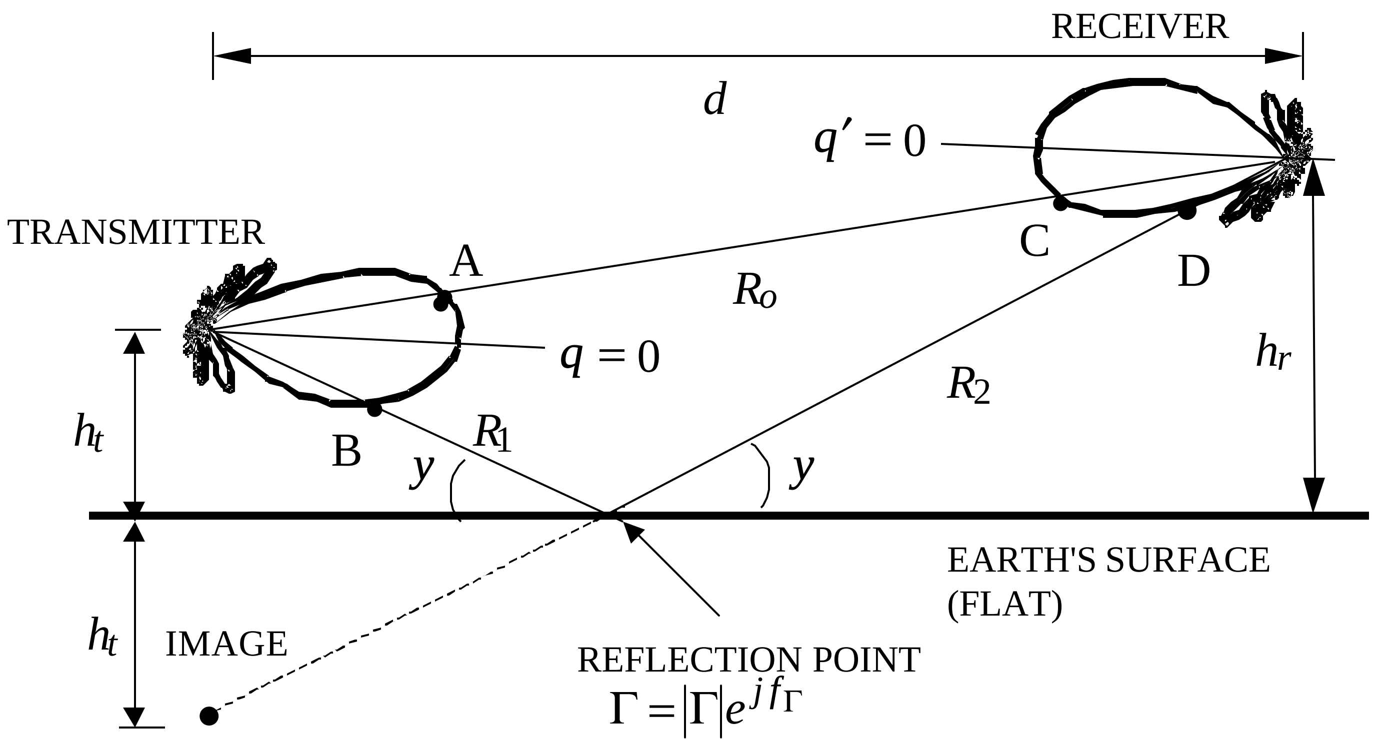
by Professor David Jenn



(version 1.6)

Multipath From a Flat Ground (1)

When both a transmitter and receiver are operating near the surface of the earth, multipath (multiple reflections) can cause fading of the signal. We first examine the reflection from the ground assuming a flat earth. These mirror like reflections that obey Snell's law are called specular reflections. The reflected wave appears to originate from an image.



Multipath From a Flat Ground (2)

Multipath parameters:

1. Reflection coefficient, $\Gamma = |\Gamma| e^{j f_\Gamma}$. For low grazing angles, $y \approx 0$, the approximation $\Gamma \approx -1$ is valid for both horizontal and vertical polarizations.
2. Transmit antenna gain: $G_t(q_A)$ for the direct wave; $G_t(q_B)$ for the reflected wave.
3. Receive antenna gain: $G_r(q_C)$ for the direct wave; $G_r(q_D)$ for the reflected wave.
4. Path difference: $\Delta R = \underbrace{(R_1 + R_2)}_{\text{REFLECTED}} - \underbrace{R_o}_{\text{DIRECT}}$

Gain is proportional to the square of the electric field intensity. For example, if G_{to} is the gain of the transmit antenna in the direction of the maximum ($q = 0$), then

$$G_t(q) = G_{to} \left| E_{t_{\text{norm}}}(q) \right|^2 \equiv G_{to} f_t(q)^2$$

where $E_{t_{\text{norm}}}$ is the normalized electric field intensity. Similarly for the receive antenna with its maximum gain in the direction $q' = 0$

$$G_r(q') = G_{ro} \left| E_{r_{\text{norm}}}(q') \right|^2 \equiv G_{ro} f_r(q')^2$$

Multipath From a Flat Ground (3)

Total field at the receiver

$$\left|E_{\text{tot}}\right| = \underbrace{E_{\text{ref}}}_{\text{REFLECTED}} + \underbrace{E_{\text{dir}}}_{\text{DIRECT}}$$

$$= \left| f_t(q_A)\, f_r(q_C)\, \frac{e^{-jkR_o}}{4p\, R_o} \overbrace{\left[1 + \Gamma \frac{f_t(q_B)\, f_r(q_D)}{f_t(q_A)\, f_r(q_C)} e^{-jk\Delta R} \right]}^{\equiv F} \right|$$

The quantity in the square brackets is the <u>path-gain factor</u> (PGF) or <u>pattern-propagation factor</u> (PPF). It relates the total field at the receiver to that of free space and takes on values $0 \le F \le 2$.

- If $F = 0$ then the direct and reflected rays cancel (destructive interference)
- If $F = 2$ the two waves add (constructive interference)

Note that if the transmitter and receiver are at approximately the same heights, close to the ground, and the antennas are pointed at each other, then $d >> h_t, h_r$ and

$$G_t(q_A) \approx G_t(q_B)$$
$$G_r(q_C) \approx G_r(q_D)$$

Multipath From a Flat Ground (4)

An approximate expression for the path difference is obtained from a series expansion:

$$R_o = \sqrt{d^2 + (h_r - h_t)^2} \approx d + \frac{1}{2}\frac{(h_r - h_t)^2}{d}$$

$$R_1 + R_2 = \sqrt{d^2 + (h_t + h_r)^2} \approx d + \frac{1}{2}\frac{(h_t + h_r)^2}{d}$$

Therefore,

$$\Delta R \approx \frac{2h_r h_t}{d}$$

and

$$|F| = \left|1 - e^{-jk2h_r h_t/d}\right| = \left|e^{jkh_r h_t/d}\left(e^{-jkh_r h_t/d} - e^{jkh_r h_t/d}\right)\right| = 2\left|\sin\left(kh_r h_t/d\right)\right|$$

The received power depends on the square of the path gain factor

$$P_r \propto |F|^2 = 4\sin^2\left(\frac{kh_t h_r}{d}\right) \approx 4\left(\frac{kh_t h_r}{d}\right)^2$$

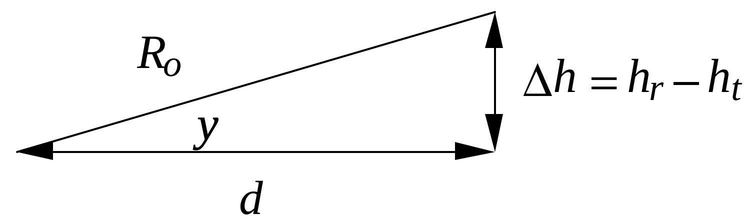
The last approximation is based on $h_r, h_t << d$ and $\Gamma \approx$ -1.

Multipath From a Flat Ground (5)

Two different forms of the argument are frequently used.

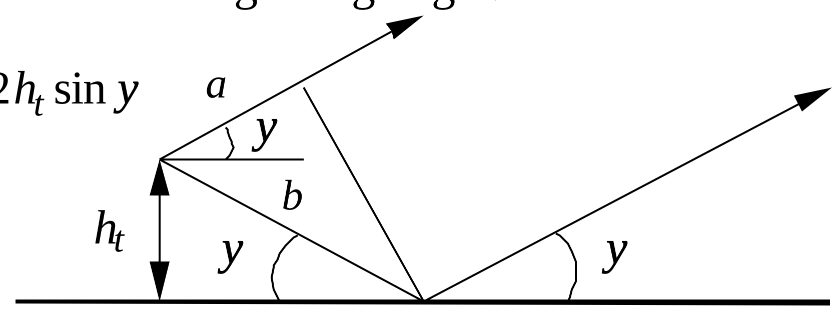
1. Assume that the transmitter is near the ground $h_t \approx 0$ and use its height as a reference. The elevation angle is y where

$$\tan y = \frac{h_r - h_t}{d} \equiv \frac{\Delta h}{d} \approx \frac{h_r}{d}$$



2. If the transmit antenna is very close to the ground, then the reflection point is very near to the transmitter and y is also the grazing angle:

$$\Delta R = b - a = 2 h_t \sin y$$



If the antenna is pointed at the horizon (i.e., its maximum is parallel to the ground) then $y \approx q_A$.

Multipath From a Flat Ground (6)

Thus with the given restrictions the PPF can be expressed in terms of elevation angle y

$$|F| = 2\sin\left(kh_t \tan y\right)$$

The PPF has minima at:

$$kh_t \tan y = np \qquad (n = 0, 1, \ldots, \infty)$$

$$\frac{2p}{l} h_t \tan y = np$$

$$\tan y = n l / h_t$$

Maxima occur at:

$$kh_t \tan y = mp/2 \qquad (m = 1, 3, 5, \ldots, \infty)$$

$$\frac{2p}{l} h_t \tan y = \frac{2n+1}{2} p \qquad (n = 0, 1, \ldots, \infty)$$

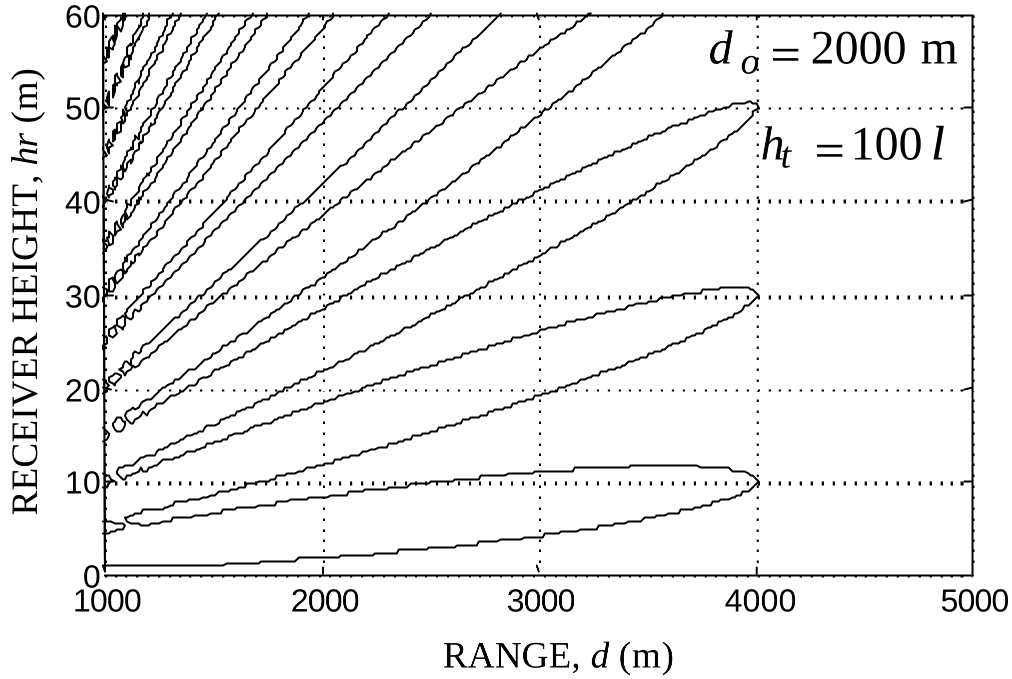
$$\tan y = \frac{(2n+1)\, l}{4h_t}$$

Plots $|F|$ are called a <u>coverage diagram</u>. The horizontal axis is usually distance and the vertical axis receiver height. (Note that because $d >> h_r$ the angle y is not directly measurable from the plot.)

Multipath From a Flat Ground (7)

<u>Coverage diagram</u>: Contour plots of $|F|$ in dB for variations in h_r and d normalized to a reference range d_o. Note that when $d = d_o$ then $E_{\text{tot}} = E_{\text{dir}}$.

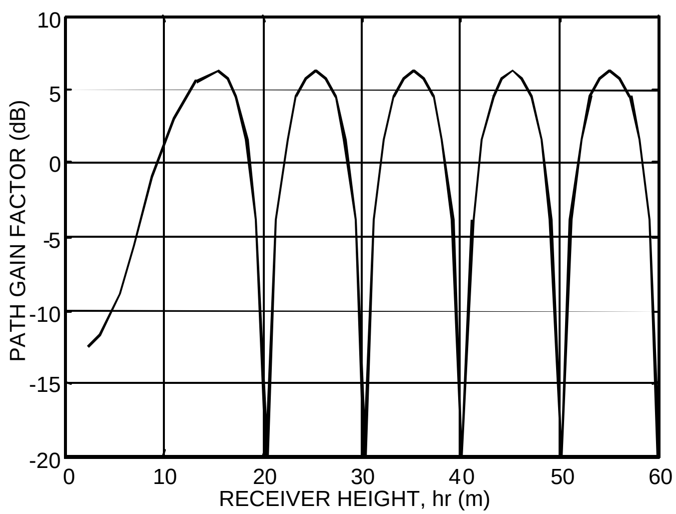
$$|F| = \left| 2\left(\frac{d_o}{d}\right)\sin\left(kh_t \tan\psi\right)\right|$$



Multipath From a Flat Ground (8)

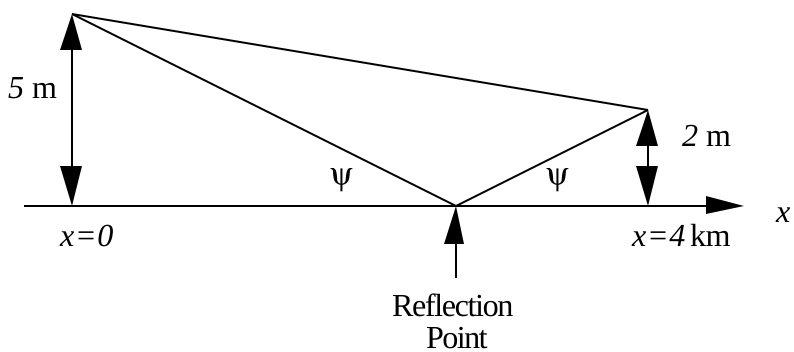
Another means of displaying the received field is a height-gain curve. It is a plot of $|F|$ in dB vs. h_r at a fixed range.

- The constructive and destructive interference as a function of height can be identified.
- At low frequencies the periodicity of the curve at low heights can be destroyed by the ground wave.
- Usually there are many reflected wave paths between the transmitter and receiver, in which case the peaks and nulls are distorted.
- This technique is often used to determine the optimum tower height for a broadcast radio antenna.



Multipath Example

A radar antenna is mounted on a 5 m mast and tracks a point target at 4 km. The target is 2 m above the surface and the wavelength is 0.2 m. (a) Find the location of the reflection point on the x axis and the grazing angle y . (b) Write an expression for the one way path gain factor $|F|$ when a reflected wave is present. Assume a reflection coefficient of $\Gamma \approx -1$.



(a) Denote the location of the reflection point by x_r and use similar triangles

$$\tan y = \frac{5}{x_r} = \frac{2}{4000 - x_r}$$

$$x_r = 2.86 \text{ km}$$

$$y = \tan^{-1}(5/2860) = 0.1^\circ$$

(b) The restrictions on the heights and distance are satisfied for the following formula

$$|F| = 2\left|\sin\left(\frac{kh_t h_r}{d}\right)\right| = 2\left|\sin\left(\frac{2p(2)(5)}{(0.2)(4000}\right)\right|$$

$$= (2)(0.785) = 0.157$$

The received power varies as $|F|^2$, thus

$$10\log\left(|F|^2\right) = -16.1 \text{ dB}$$

The received power is 16.1 dB below the free space value.

Field Intensity From the ERP

The product $P_t G_t$ is called the effective radiated power (ERP, or sometimes the effective isotropic radiated power, EIRP). We can relate the ERP to the electric field intensity as follows:

The Poynting vector for a TEM wave:

$$\vec{W} = \mathrm{Re}\left\{\vec{E} \times \vec{H}^*\right\} = \frac{\left|\vec{E}_{\mathrm{dir}}\right|^2}{h_o}$$

For the direct path:

$$\left|\vec{W}\right| = \frac{P_t G_t}{4pR_o^2}$$

Equate the two expressions: (and note that $h_o \approx 120p$)

$$\frac{\left|\vec{E}_{\mathrm{dir}}\right|^2}{h_o} = \frac{P_t G_t}{4pR_o^2} \quad \Rightarrow \quad \left|\vec{E}_{\mathrm{dir}}\right| = \frac{\sqrt{30 P_t G_t}}{d} \equiv \frac{E_o}{d}$$

where E_o is called the unattenuated field intensity at unit distance.

Wave Reflection at the Earth's Surface (1)

Fresnel reflection coefficients hold when:

1. the Earth's surface is locally flat in the vicinity of the reflection point
2. the surface is smooth (height of irregularities << l)

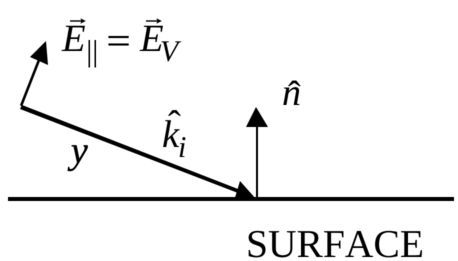
Traditionally, the grazing angle $y = 90^\circ - q_i$ is used. The grazing angle is usually very small ($y < 1^\circ$). The complex dielectric constant is expressed as

$$e_c = e_r e_o - j\frac{s}{w} = e_o\left(e_r - j\frac{s}{e_o w}\right) \equiv e_o \underbrace{\left(e_r - jc\right)}_{e_{rc}}, \text{ where } c = \frac{s}{w e_o}$$

Near the surface the reference for polarization is horizontal and vertical.

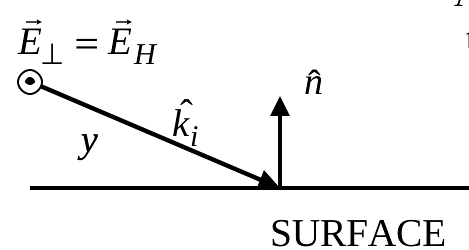
VERTICAL POL

Also called transverse magnetic (TM) pol



HORIZONTAL POL

Also called transverse electric (TE) pol



Wave Reflection at the Earth's Surface (2)

Reflection coefficients for horizontal and vertical polarization:

$$-\Gamma_{\parallel} \equiv R_V = \frac{(e_r - jc)\sin y - \sqrt{(e_r - jc) - \cos^2 y}}{(e_r - jc)\sin y + \sqrt{(e_r - jc) - \cos^2 y}}$$

$$\Gamma_{\perp} \equiv R_H = \frac{\sin y - \sqrt{(e_r - jc) - \cos^2 y}}{\sin y + \sqrt{(e_r - jc) - \cos^2 y}}$$

For vertical polarization the phenomenon of <u>total reflection</u> can occur. This occurs when the wave is incident from a "more dense medium" to a "less dense" medium. From Snell's law, assuming $m_r = 1$,

$$\sin q_i = \sin q_r = \sqrt{(e_r - jc)m_r}\,\sin q_t \quad \underset{m_r = 1}{\Rightarrow} \quad \sin q_t = \frac{\sin q_i}{\sqrt{e_r - jc}}$$

There is no physical reason why $\sin q_t$ cannot be greater than 1. Let q_t be complex, $q_t = \frac{p}{2} + jq$, where q is real. Therefore,

$$\sin q_t = \sin\left(\frac{p}{2} + jq\right) = \cos(jq) = \cosh q \quad \text{and} \quad \cos q_t = -j\sin(jq) = -j\sinh q$$

Wave Reflection at the Earth’s Surface (3)

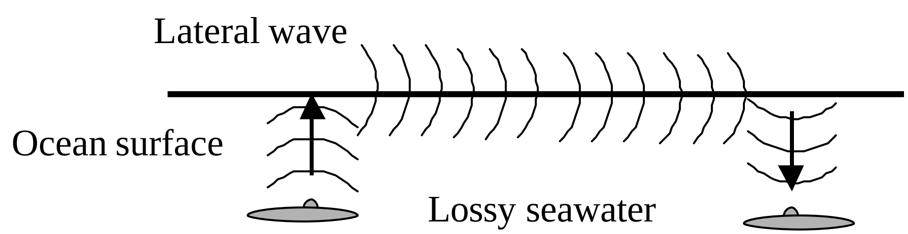
Snell’s law becomes

$$\sin q_t = \cosh q = \frac{\sin q_i}{\sqrt{e_r - jc}} \quad \text{and} \quad \cos q_t = \sqrt{1 - \sin^2 q_t} = \sqrt{1 - \cosh^2 q} = \sinh q$$

Reflection coefficient for vertical polarization:

$$\Gamma_{\|} \equiv -R_V = \frac{jh \sinh q + h_o \cos q_i}{jh \sinh q - h_o \cos q_i}$$

where $h = \sqrt{\dfrac{m_o}{e_o(e_r - jc)}}$. Note that $\left|\Gamma_{\|}\right| = 1$ and therefore <u>all of the power flow is along the surface</u>. The wave decays exponentially with distance into the less dense medium. An example where this might occur is communication between submerged submarines near the surface. The wave guided by the surface is called a <u>lateral wave</u>.

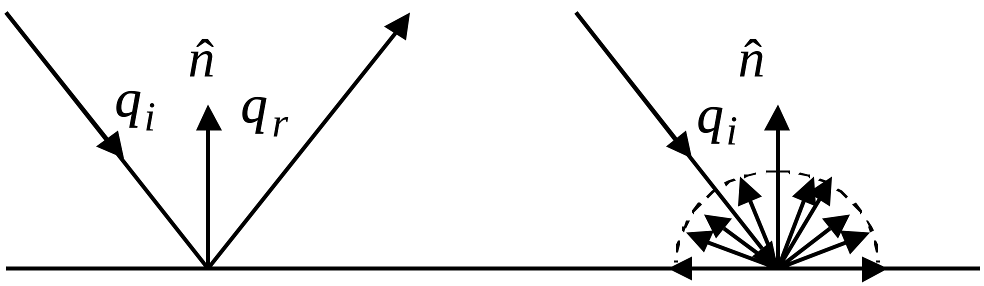


Rough Surface Reflection (1)

When waves are reflected by the Earth, the deviation from a perfectly flat surface causes waves to be scattered diffusely. An ideal diffuse surface scatters (reflects) uniformly in all directions independent of the angle of incidence. Snell's law does not apply to diffuse reflection.

At radio and microwave frequencies surfaces usually fall in between these two limiting cases. Both specular and diffuse components are present.

The traditional measure of surface roughness is the Rayleigh criterion, which is a measure of the phase error introduced by the surface deviations, relative to phase of the reflection from the mean surface.



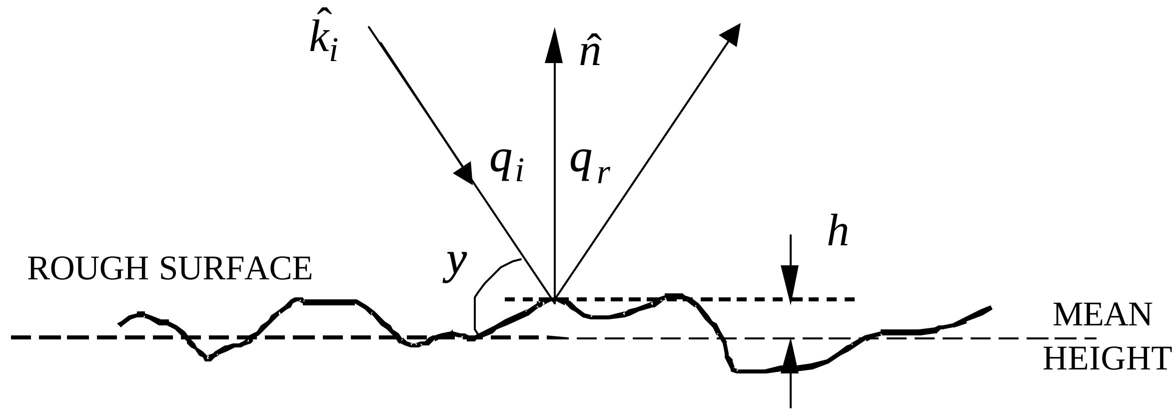
Perfectly Flat Surface (specular reflection satisfies Snell's law)

Perfectly diffuse surface (rays reflected in all directions)

Rough Surface Reflection (2)

The phase error due to the displacement of the reflection point a distance *h* from the mean surface is

$$g = k\Delta R = 2kh \sin y$$



The 2 is because the path difference occurs both on incidence and reflection. The Rayleigh condition (arbitrarily) chooses $p/2$ as the limiting value of the phase difference:

$$h \sin y < l/8$$

For small surface deviations, the diffuse component is ignored and the specular reflection coefficient is reduced. An approximate formula for the reduction of the Fresnel reflection coefficient for a rough surface is

$$|\Gamma| \rightarrow |\Gamma| \sqrt{\frac{1 + \left(\frac{1}{2} g^2\right)}{1 + 2.35\left(\frac{1}{2} g^2\right) + 2p\left(\frac{1}{2} g^2\right)^2}}$$

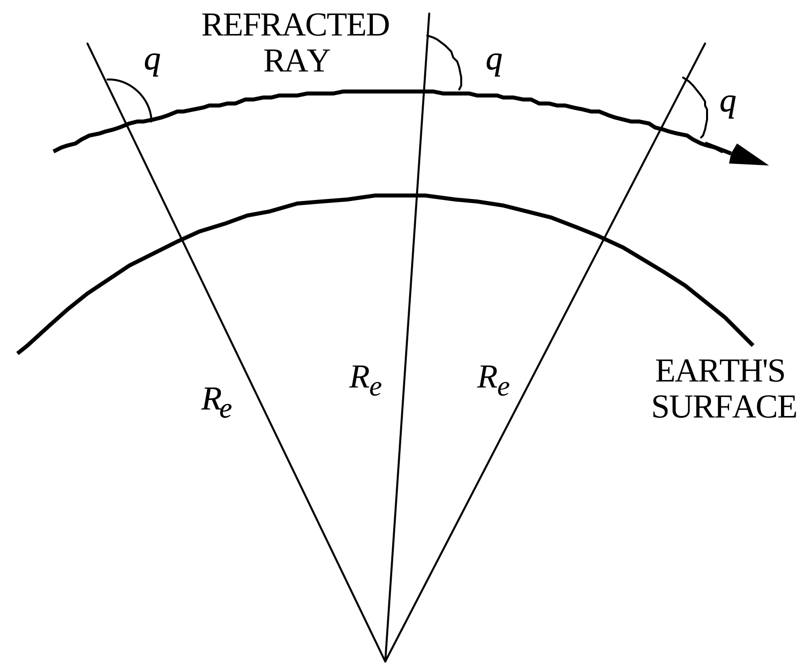
Atmospheric Refraction (1)

Refraction by the lower atmosphere causes waves to be bent back towards the earth's surface. The ray trajectory is described by the equation: $n R_e \sin q = \text{CONSTANT}$

Two ways of expressing the index of refraction $n \left(= \sqrt{e_r}\right)$ in the troposphere:

1. $n = 1 + c\, r / r_{\text{SL}}$ +HUMIDITY TERM

R_e = 6378 km = earth radius
$c \approx 0.00029 =$ Gladstone-Dale constant
r, r_{SL} = mass densities at altitude and sea level

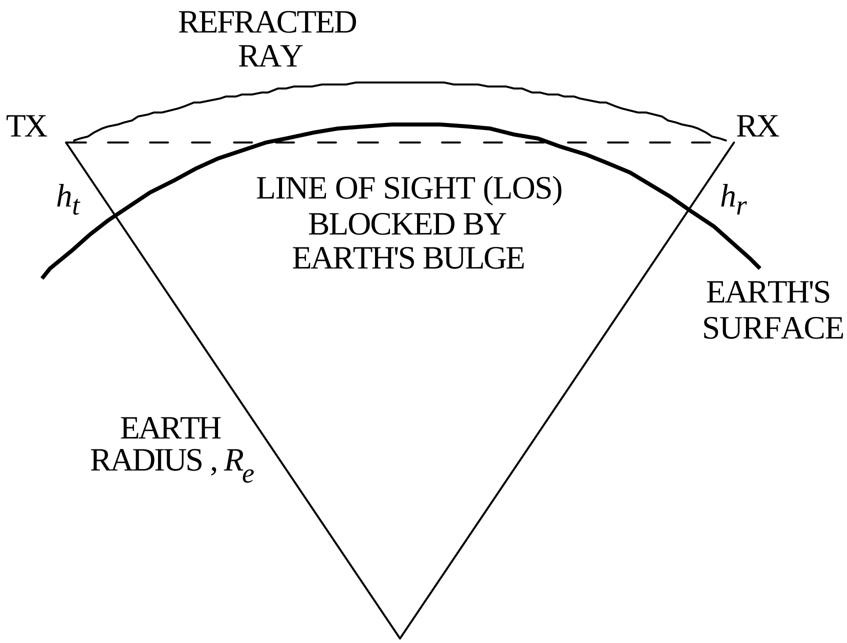


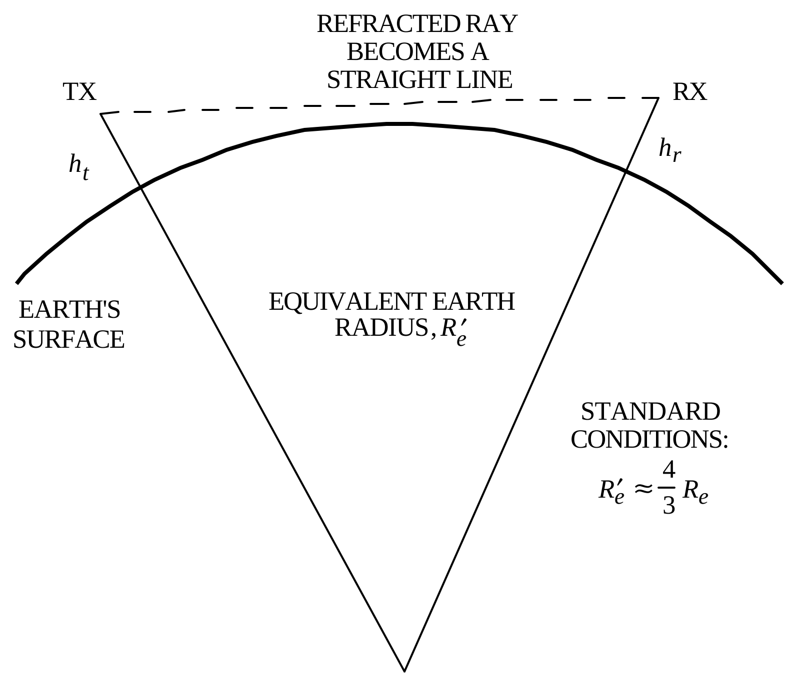
2. $n = \dfrac{77.6}{\text{T}}\left(p + 4{,}810\, e / \text{T}\right) 10^{-6} - 1$

$p =$ air pressure (millibars)
T = temperature (K)
$e =$ partial pressure of water vapor (millibars)

Atmospheric Refraction (2)

Refraction of a wave can provide a significant level of transmission over the horizon. A bent refracted ray can be represented by a straight ray if an equivalent earth radius R_e' is used. For most atmospheric conditions $R_e' = 4R_e/3 \approx 8500$ km.



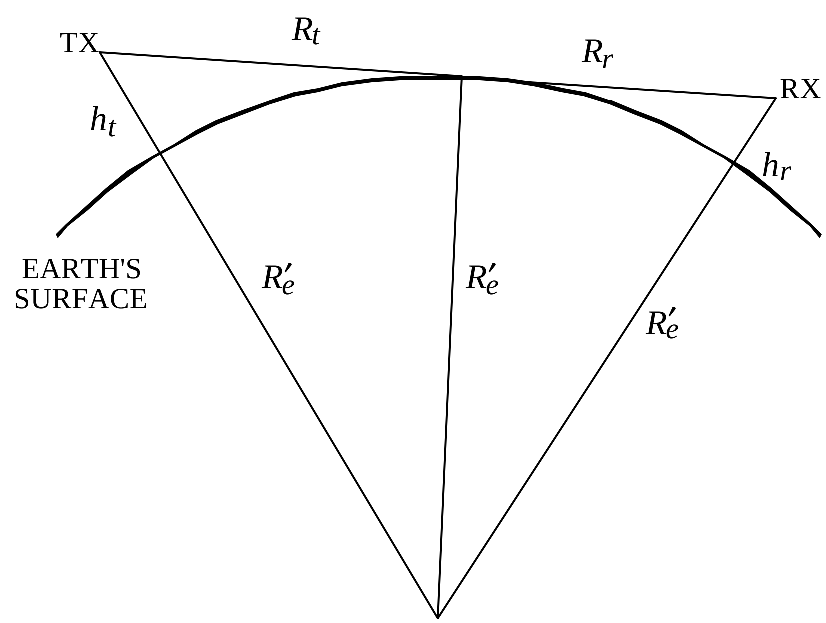


Atmospheric Refraction (3)

Distance from the transmit antenna to the horizon is $R_t = \sqrt{(R'_e + h_t)^2 - (R'_e)^2}$ but $R'_e >> h_t$ so that $R_t \approx \sqrt{2R'_e h_t}$. Similarly $R_r \approx \sqrt{2R'_e h_r}$. The radar horizon (or radio horizon) is the sum

$$R_{RH} \approx \sqrt{2R'_e h_t} + \sqrt{2R'_e h_r}$$

Example: A missile is flying 15 m above the ocean towards a ground based radar. What is the approximate range that the missile can be detected assuming standard atmospheric conditions?

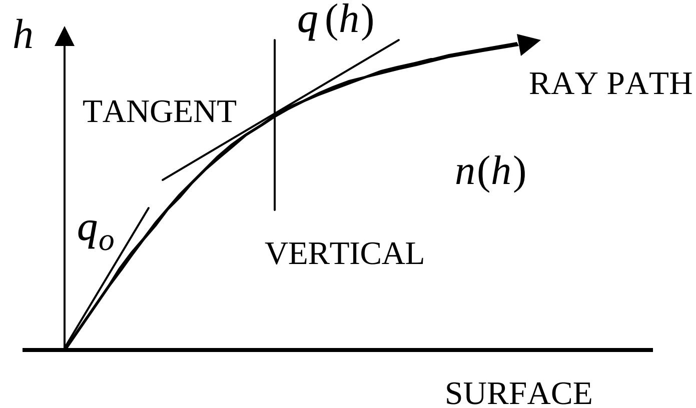


Using $h_t = 0$ and $h_r = 15$ gives a radar horizon of

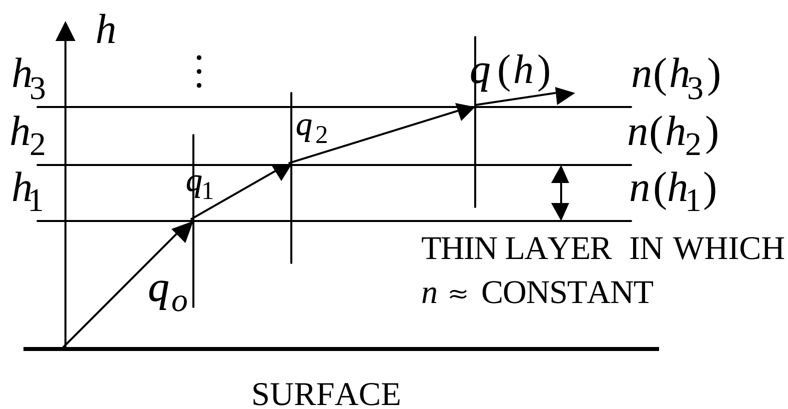
$$\begin{aligned} R_{RH} &\approx \sqrt{2R'_e h_r} \\ &\approx \sqrt{(2)(8500 \times 10^3)(15)} \\ &\approx 16 \text{ km} \end{aligned}$$

Atmospheric Refraction (4)

Derivation of the equivalent Earth radius



Break up the atmosphere into thin horizontal layers. Snell's law must hold at the boundary between each layer, $\sqrt{e(h)}\sin[q(h)] = \sqrt{e_o}\sin q_o$



Atmospheric Refraction (5)

Snell's law for spherically stratified layers is,

$$\underbrace{R_e\sqrt{e_o}\sin q_o}_{\substack{\text{AT THE}\\ \text{SURFACE}}} = \underbrace{R\sqrt{e(R)}\sin[q(R)]}_{\substack{\text{AT RADIUS}\\ R=R_e+h}}$$

Using the grazing angle, and assuming that $e(h)$ varies linearly with h

$$R_e\sqrt{e_o}\cos y_o = (R_e+h)\left\{\sqrt{e_o}+h\frac{d}{dh}\sqrt{e(h)}\right\}\cos[y(h)]$$

Expand and rearrange

$$R_e\sqrt{e_o}\{\cos y_o-\cos[y(h)]\}=\left\{\sqrt{e_o}+R_e\frac{d}{dh}\sqrt{e(h)}\right\}h\cos[y(h)]+h^2\frac{d}{dh}\sqrt{e(h)}\,\cos[y(h)]$$

If $h \ll R_e$ then the last term can be dropped, and since y is small, $\cos y \approx 1+y^2/2$

$$[y(h)]^2 \approx y_o^2+\frac{2h}{R_e}\left[1+\frac{R_e}{\sqrt{e_o}}\frac{d}{dh}\sqrt{e(h)}\right]$$

The second term is due to the inhomogeneous index of refraction with altitude.

Atmospheric Refraction (6)

Define a constant k such that

$$[y(h)]^2 \approx y_o^2 + \frac{2h}{kR_e} = y_o^2 + \frac{2h}{R_e'} \qquad \text{where} \qquad k = \left[1 + \frac{R_e}{\sqrt{e_o}} \frac{d}{dh} \sqrt{e(h)}\right]^{-1}$$

$R_e' = kR_e$ is the effective (equivalent) Earth radius. If R_e' is used as the Earth radius then rays can be drawn as straight lines. This is the radius that would produce the same geometrical relationship between the source of the ray and the receiver near the Earth's surface, assuming a constant index of refraction. The restrictions on the model are:

1. Ray paths are nearly horizontal
2. $\sqrt{e(h)}$ versus h is linear over the range of heights considered

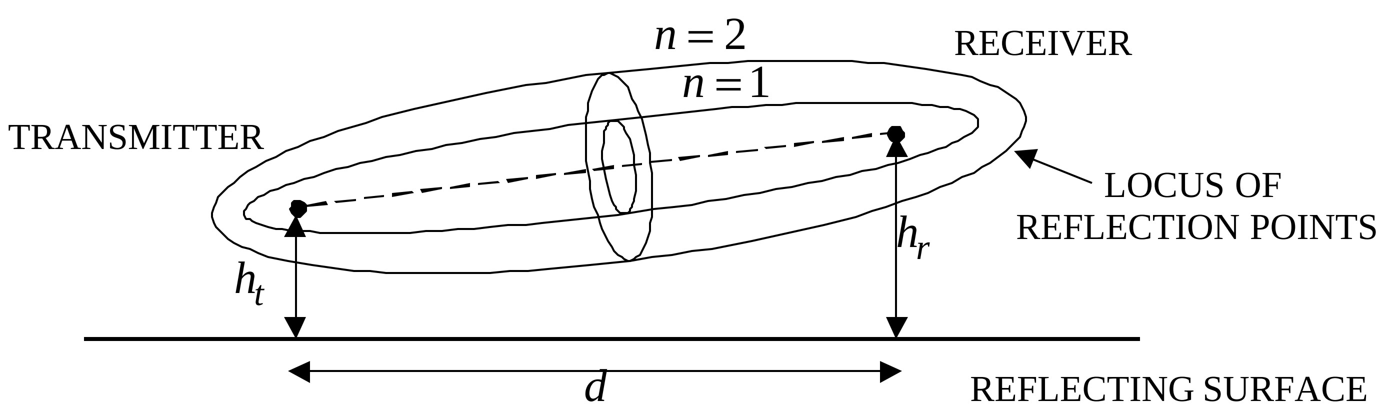
Under standard (normal) atmospheric conditions, $k \approx 4/3$. That is, the radius of the Earth is approximately $R_e' = \left(\frac{4}{3}\right) 6378\,\text{km} \approx 8500\,\text{km}$. This is commonly referred to as "the four-thirds Earth approximation."

Fresnel Zones (1)

For the direct path phase to differ from the reflected path phase by an integer multiple of 180° the paths must differ by integer multiples of $l\,/\,2$

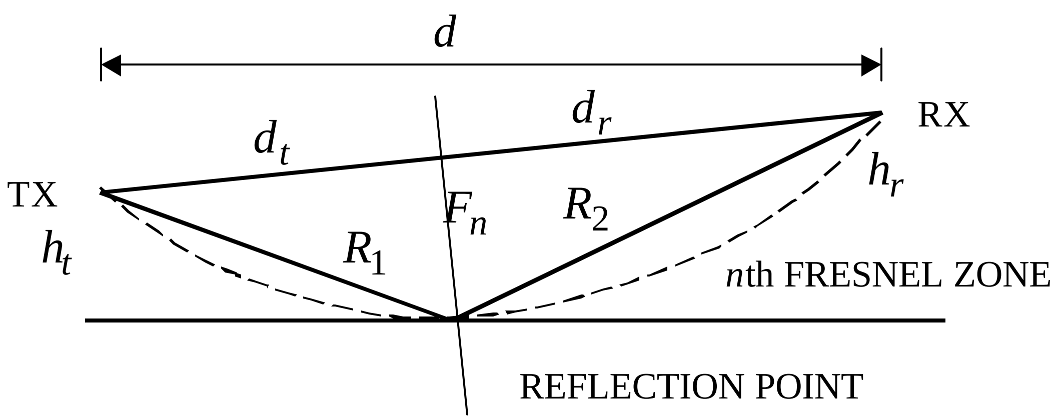
$$\Delta R = n\,l\,/\,2 \qquad (n = 0,1,\ldots)$$

The collection of points at which reflection would produce an excess path length of $n\,l\,/\,2$ is called the <u>nth Fresnel zone</u>. In three dimensions the surfaces are ellipsoids centered on the direct path between the transmitter and receiver.



Fresnel Zones (2)

A slice of the vertical plane gives the following geometry



For the reflection coefficient $\Gamma = |\Gamma| e^{jp} = -|\Gamma|$:

- If <u>n is even</u> the two paths are out of phase and the received signal is a <u>minimum</u>
- If <u>n is odd</u> the two paths are in phase and the received signal is a <u>maximum</u>

Because the LOS is nearly horizontal $R_o \approx d$ and therefore $R_o = d_t + d_r \approx d$. For the nth Fresnel zone $R_1 + R_2 = d + nl/2$.

Fresnel Zones (3)

The radius of the nth Fresnel zone is

$$F_n = \sqrt{\frac{n\, l\, d_t d_r}{d}}$$

or, if the distances are in miles, then

$$F_n = 72.1\sqrt{\frac{n d_t d_r}{f_{\mathrm{GHz}} d}} \text{ (feet)}$$

Transmission path design: the objective is to find transmitter and receiver locations and heights that give signal maxima. In general:

1. reflection points should not lie on even Fresnel zones
2. the LOS should clear all obstacles by $0.6F_1$, which essentially gives free space transmission

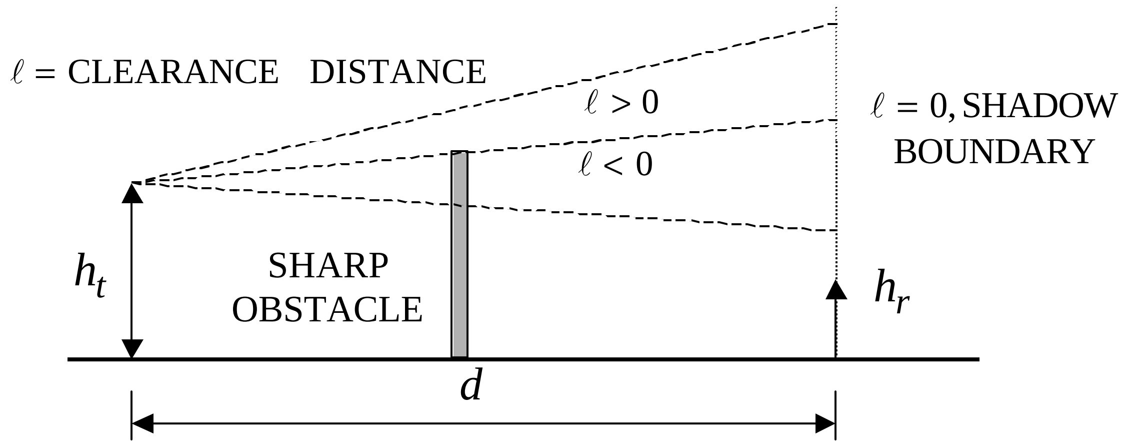
The significance of $0.6F_1$ is illustrated by examining two canonical problems:

(1) knife edge diffraction and
(2) smooth sphere diffraction.

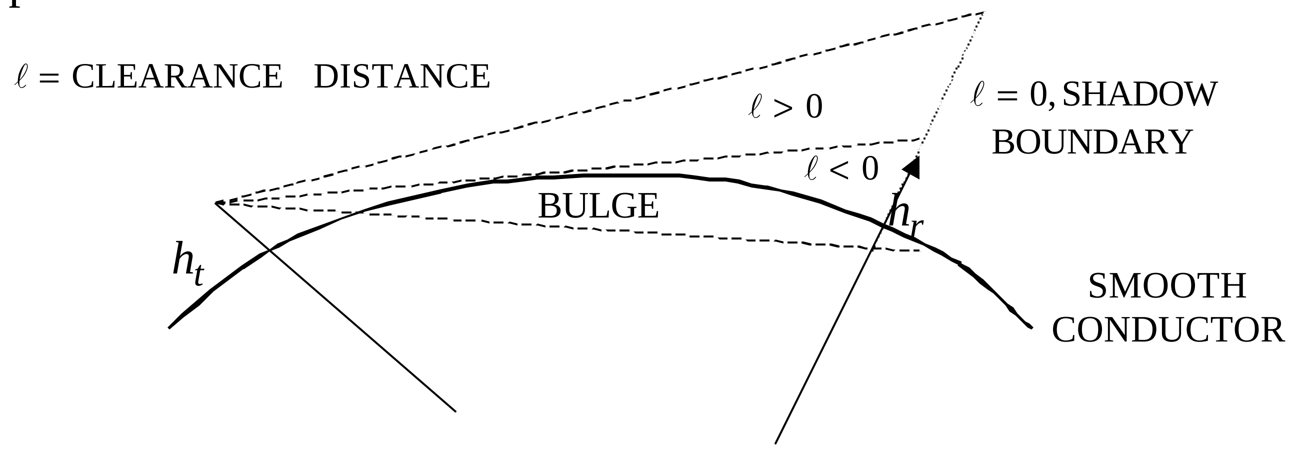
Conversions: 0.0254 m = 1 in; 12 in = 1 ft; 3.3 ft = 1 m; 5280 ft = 1 mi; 1 km = 0.62 mi

Diffraction (1)

Knife edge diffraction

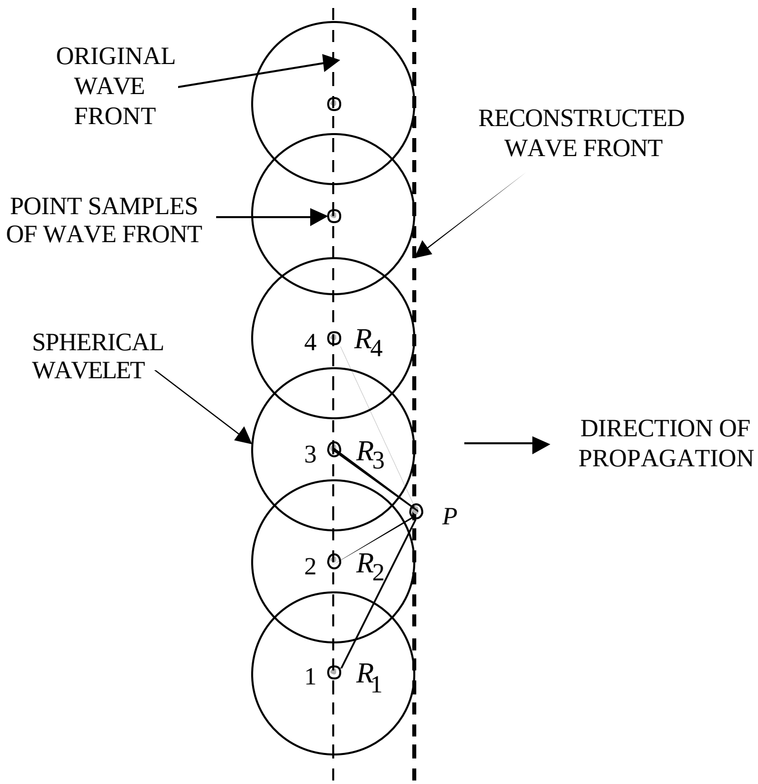


Smooth sphere diffraction



Huygen's Principle (1)

<u>Huygen's principle</u> states that any wavefront can be decomposed into a collection of point sources. New wavefronts can be constructed from the combined "spherical wavelets" from the point sources of the old wavefront.

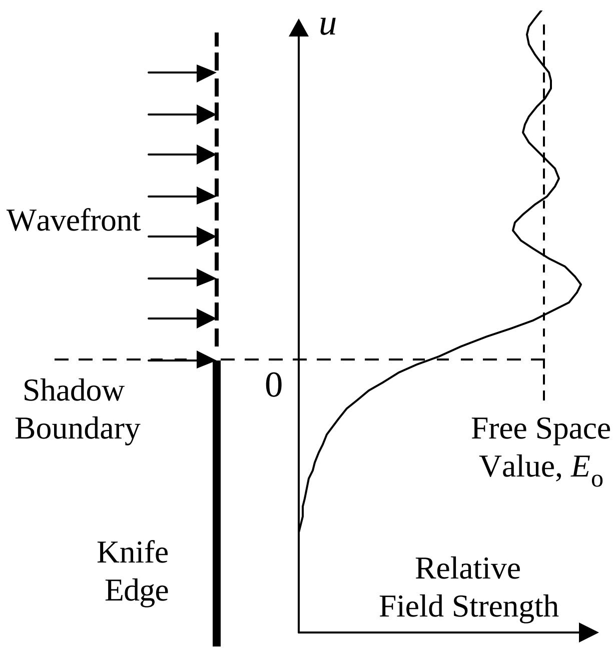


$$E(P) \sim \sum_{n=-\infty}^{\infty} \frac{e^{-jkR_n}}{R_n} \quad \rightarrow \quad \int_{-\infty}^{\infty} \frac{e^{-jkR_P}}{R_P}$$

where R is the distance from a wavelet source to the observation point, P. Sources closest to P will contribute most to the field

Knife Edge Diffraction (1)

The perturbation of a plane wave due to the presence of a knife edge can be examined using Huygen's principle. The edge blocks the spherical wavelets below the shadow boundary. The electric field decays to zero deep in the shadow (far from the edge). Far from the edge in the lit region, the plane wave is not disturbed.



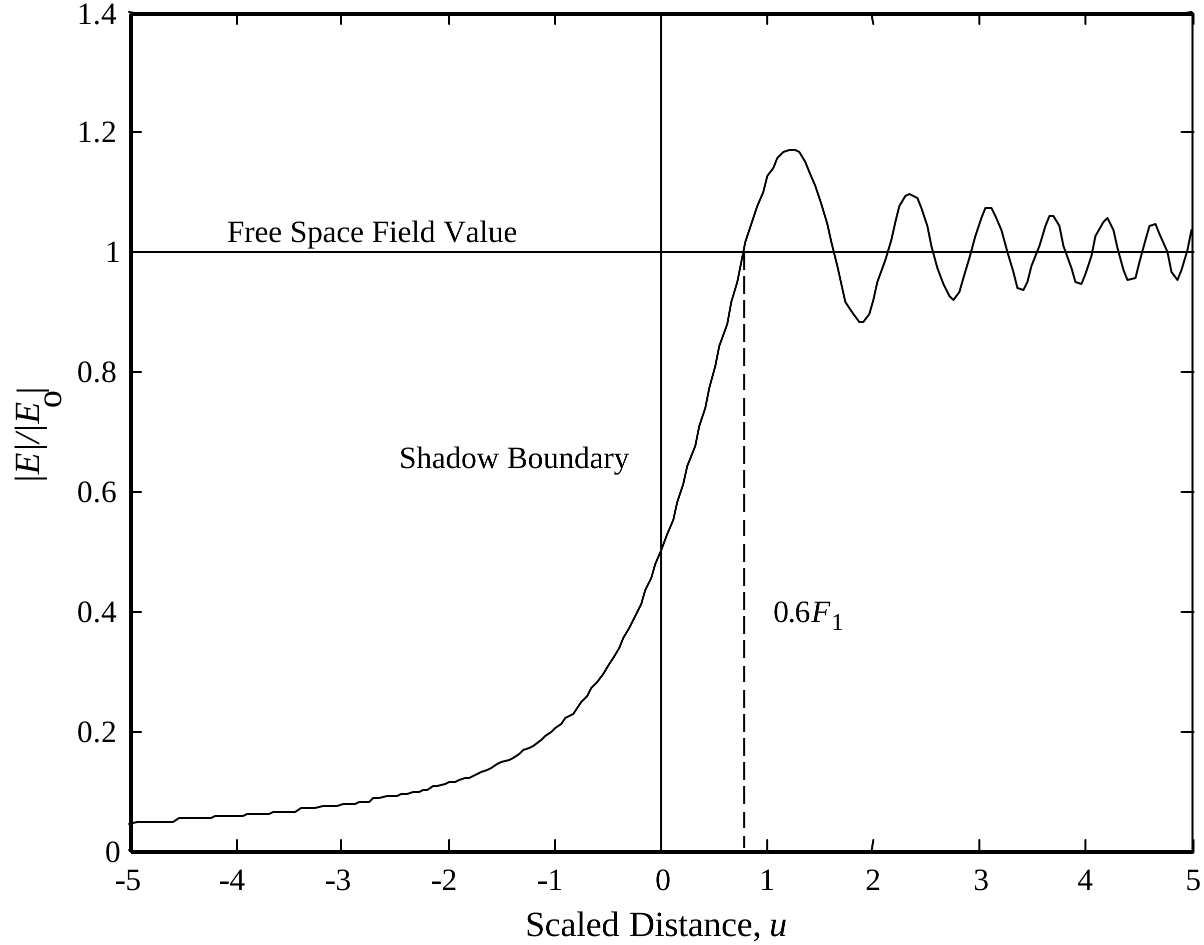
Based on Huygen's principle, the normalized electric field intensity is given by

$$\frac{E}{E_o} = \frac{1}{2} + \frac{1+j}{2} \int_0^{-u} \exp(-jpa^2/2)\, da$$

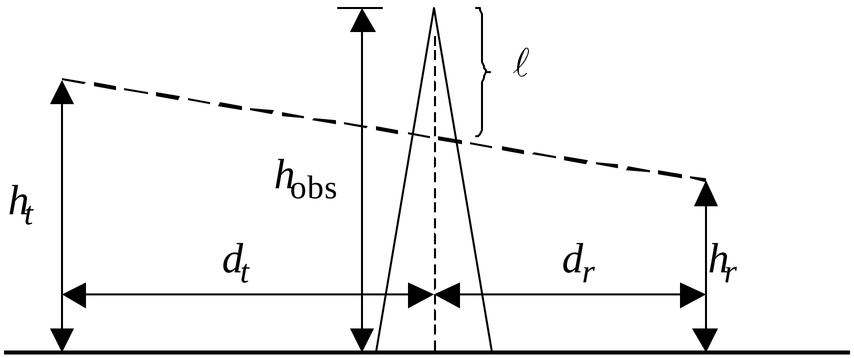
where u is a scaled distance parameter. The shadow boundary is at $u = 0$.

Knife Edge Diffraction (2)

A plot of $\left|E / E_o\right|$ shows that at $0.6F_1$ the free space (direct path) value is obtained.



Knife Edge Diffraction Example

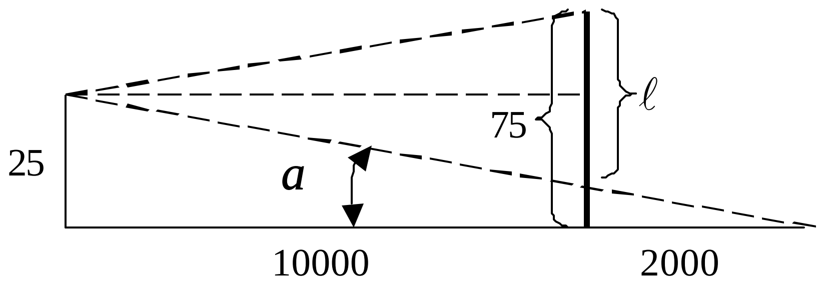


Scaled distance parameter:

$$u = \ell \sqrt{\frac{2d}{l\, d_t d_r}}$$

where $\ell < 0$ (i.e., u negative) when the obstacle extends above the direct path.

Example: $h_t = 50$ m, $h_r = 25$ m, $d_t = 10$ km, $d_r = 2$ km, $h_{\text{obs}} = 100$ m, f = 900 MHz



$$\frac{75 - \ell}{2000} = \frac{25}{12000} \quad \rightarrow \quad \ell = 75 - 4.167 = 70.8 \text{ m}$$

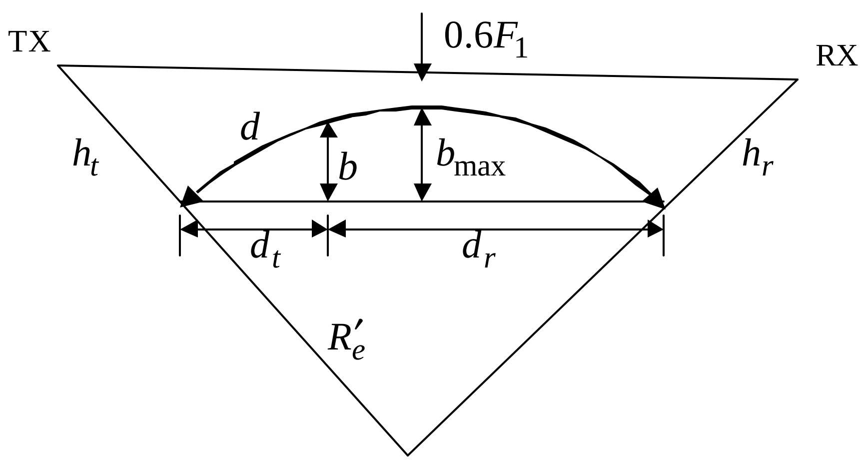
$$u = -70.8 \sqrt{\frac{24 \times 10^3}{(0.33) 24 \times 10^6}} = -4.27$$

From the diffraction plot for $u = -4.27$, $\left| E / E_{\text{o}} \right| \approx 0.053 \quad \rightarrow \quad 25.5$ dB loss

Path Clearance Example

Consider a 30 mile point-to-point communication link over the ocean. The frequency of operation is 5 GHz and the antennas are at the same height. Find the lowest height that provides the same field strength as in free space. Assume standard atmospheric conditions

The geometry is shown below (distorted scale). The <u>bulge factor</u> (in feet) is given approximately by $b = \frac{d_t d_r}{1.5k}$, where d_t and d_r are in miles.



The maximum bulge occurs at the midpoint.

$$d \approx d_t + d_r$$

$$b_{\max} = \frac{(15)(15)}{(1.5)(4/3)} = 112.5\,\text{ft}$$

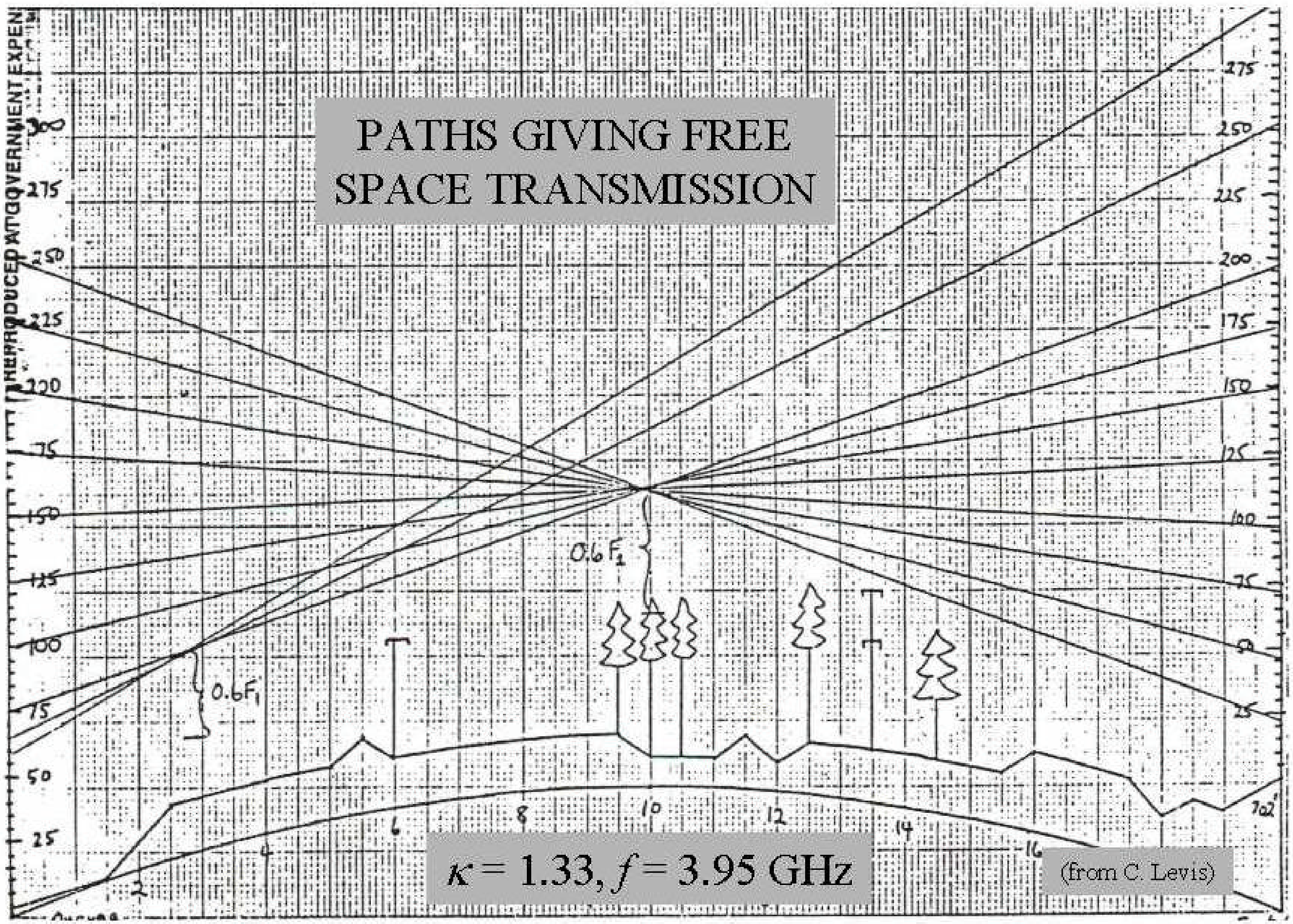
$$F_n = 72.1\sqrt{\frac{n d_t d_r}{f_{\text{GHz}} d}}\ \text{ft}$$

$$0.6F_1 = 53\,\text{ft}$$

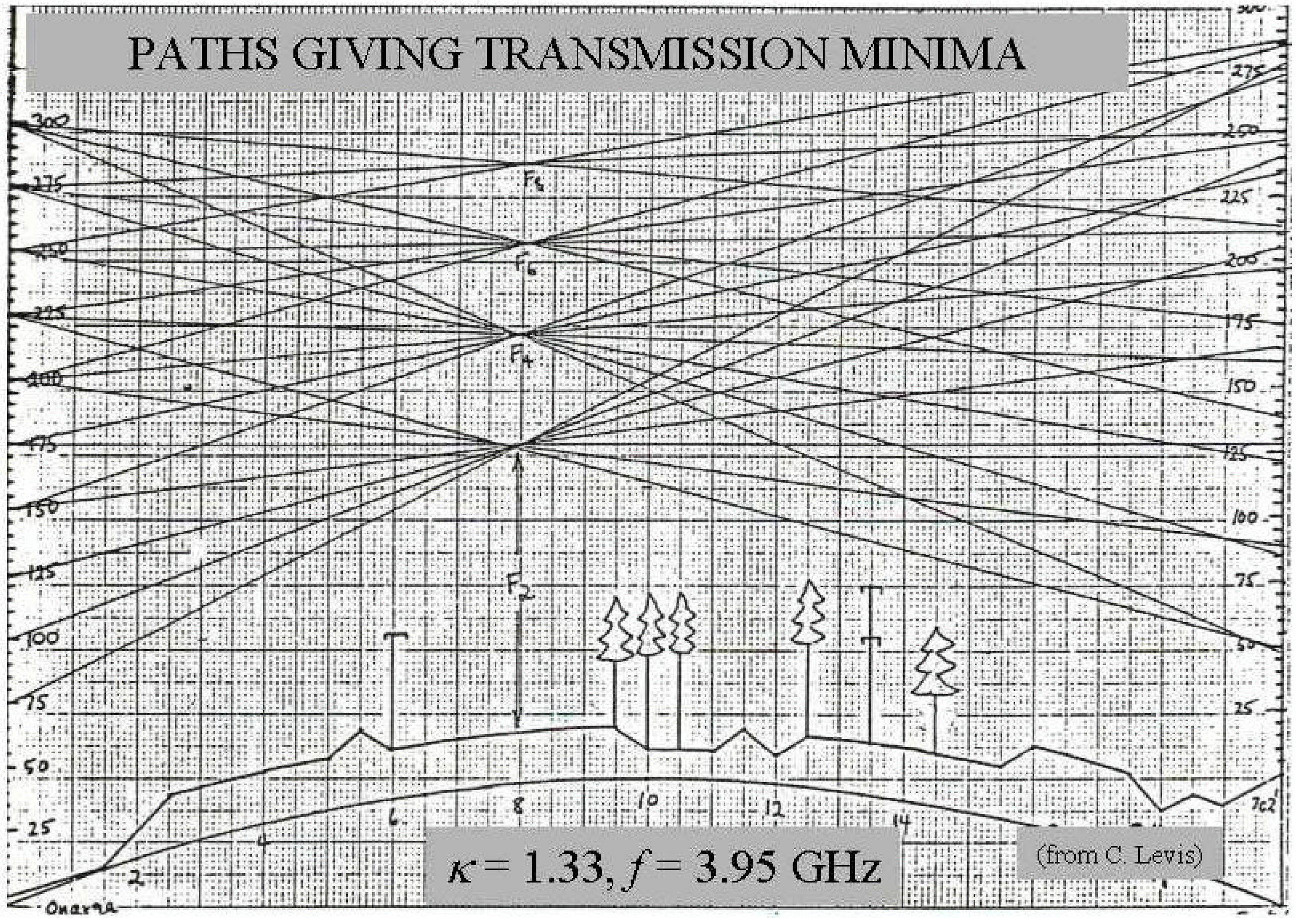
Compute the minimum antenna height:

$$h = b_{\max} + 0.6F_1 = 112.5 + 53 = 165\,\text{ft}$$

Example of Link Design (1)



Example of Link Design (2)



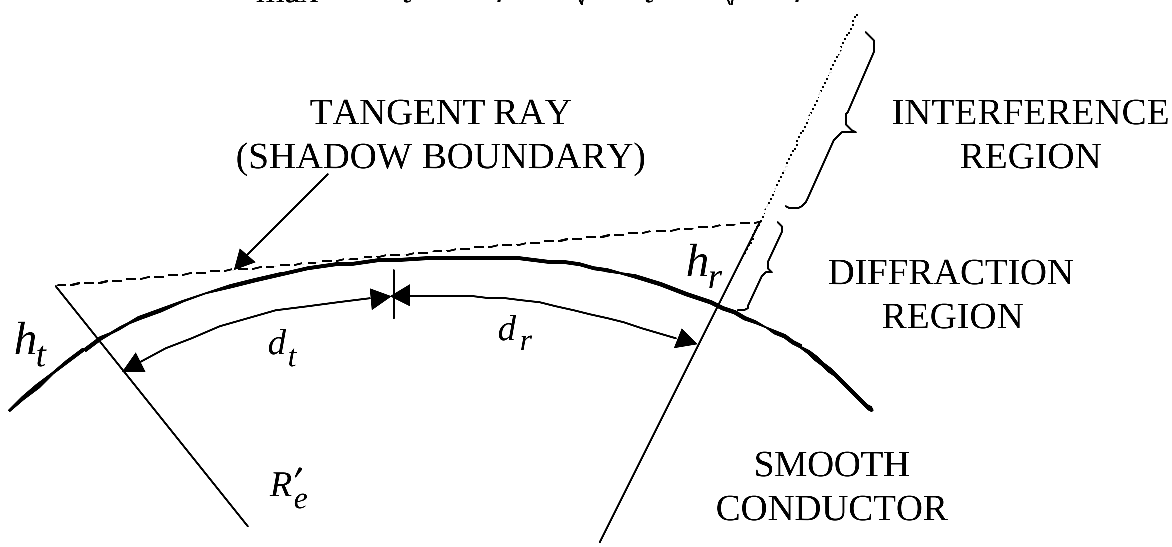
Antennas Over a Spherical Earth

When the transmitter to receiver distance becomes too large the flat Earth approximation is no longer accurate. The curvature of the surface causes:

1. divergence of the power in the reflected wave in the interference region
2. diffracted wave in the shadow region (note that this is not the same as a ground wave)

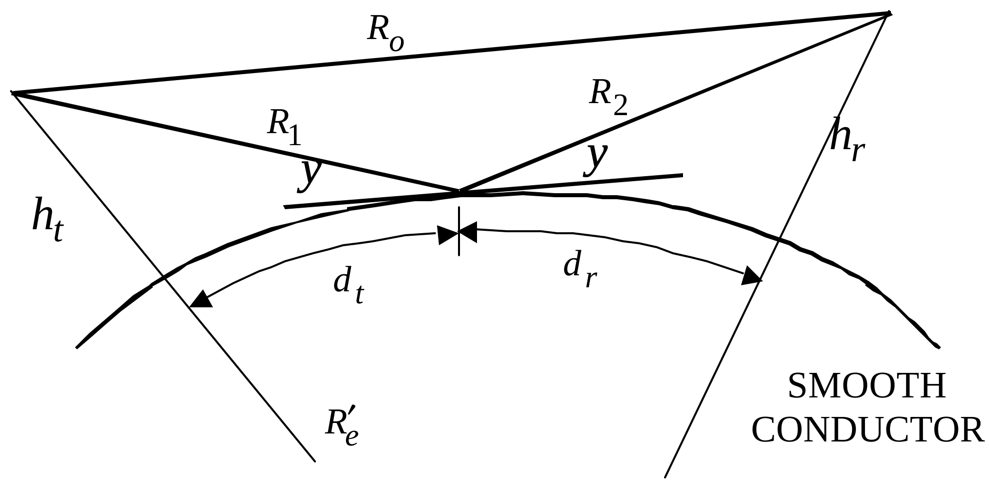
The distance to the horizon is $d_t = R_{RH} \approx \sqrt{2R'_e h_t}$ or, if h_t is in feet, $d_t \approx \sqrt{2h_t}$ miles. The maximum LOS distance between the transmit and receive antennas is

$$d_{\max} = d_t + d_r \approx \sqrt{2h_t} + \sqrt{2h_r} \quad \text{(miles)}$$



Interference Region Formulas (1)

Interference region formulas



The path-gain factor is given by

$$|F| = \left|1 + |\Gamma| e^{j f_\Gamma} e^{-jk\Delta R} \sqrt{D}\right|$$

where D is the divergence factor (power) and $\Delta R = R_1 + R_2 - R_o$.

Interference Region Formulas (2)

Approximate formulas[1] for the interference region:

$$|F| = \left\{ \left(1 + |\Gamma|\sqrt{D}\right)^2 - 4|\Gamma|\sqrt{D}\,\sin^2\left[\frac{f_\Gamma - k\Delta R}{2}\right] \right\}^{\frac{1}{2}}$$

where

$$\Delta R = \frac{2h_1 h_2}{d} J(S,T), \quad \tan y = \frac{h_1 + h_2}{d} K(S,T), \quad D = \left[1 + \frac{4 S_1 S_2^2 T}{S(1 - S_2^2)(1+T)}\right]^{-1} \text{ (power)}$$

$$S_1 = \frac{d_1}{\sqrt{2R_e' h_1}}, \quad S_2 = \frac{d_2}{\sqrt{2R_e' h_2}} \quad \text{where } h_1 \text{ is the smallest of either } h_t \text{ or } h_r$$

$$S = \frac{d}{\sqrt{2R_e' h_1} + \sqrt{2R_e' h_2}} = \frac{S_1 T + S_2}{1+T}, \quad T = \sqrt{h_1 / h_2} \quad (< 1 \text{ since } h_1 < h_2)$$

$$J(S,T) = (1 - S_1^2)(1 - S_2^2), \text{ and } K(S,T) = \frac{(1 - S_1^2) + T^2 (1 - S_2^2)}{1 + T^2}$$

[1]D. E. Kerr, *Propagation of Short Radio Waves*, Radiation Laboratory Series, McGraw-Hill, 1951 (the formulas have been reprinted in many other books including R. E. Collin, *Antennas and Radiowave Propagation*, McGraw-Hill, 1985).

Interference Region Formulas (3)

The distances can be computed from $d = d_1 + d_2$ and

$$d_1 = \frac{d}{2} + p\cos\left(\frac{\Phi + p}{3}\right), \quad \Phi = \cos^{-1}\left(\frac{2R_e'(h_1 - h_2)d}{p^3}\right),$$

and

$$p = \frac{2}{\sqrt{3}}\left[R_e'(h_1 + h_2) + \frac{d^2}{4}\right]^{1/2}$$

Another form for the phase difference is

$$k\Delta R = \frac{2kh_1h_2}{d}(1 - S_1^2)(1 - S_2^2) = nzp$$

where

$$n = \frac{4h_1^{3/2}}{l\sqrt{2R_e'}} = \frac{h_1^{3/2}}{1030\,l}, \quad z = \frac{h_2 / h_1}{d / d_{RH}}(1 - S_1^2)(1 - S_2^2),$$

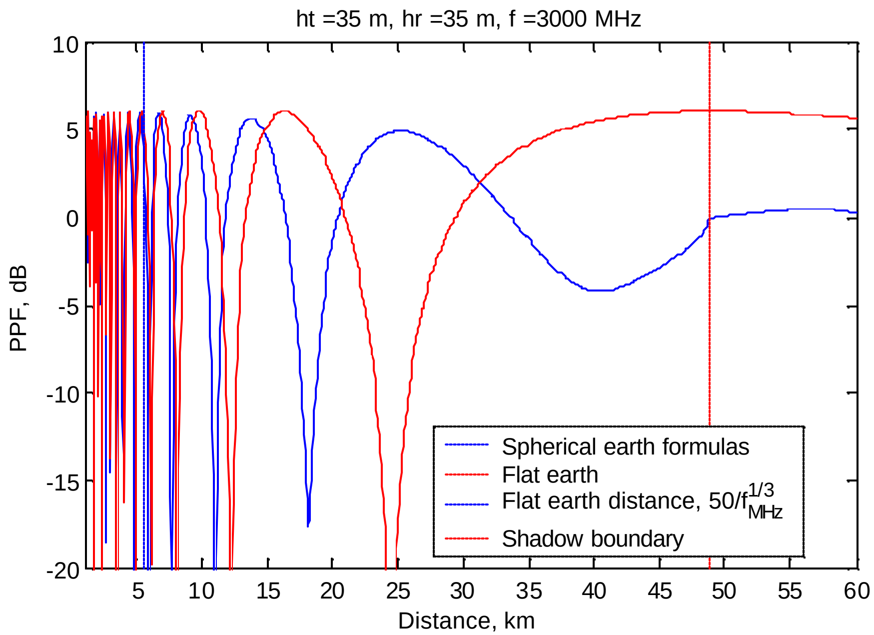
and $d_{RH} = \sqrt{2R_e'h_1}$ (distance to the radio horizon).

Interference Region Formulas (4)

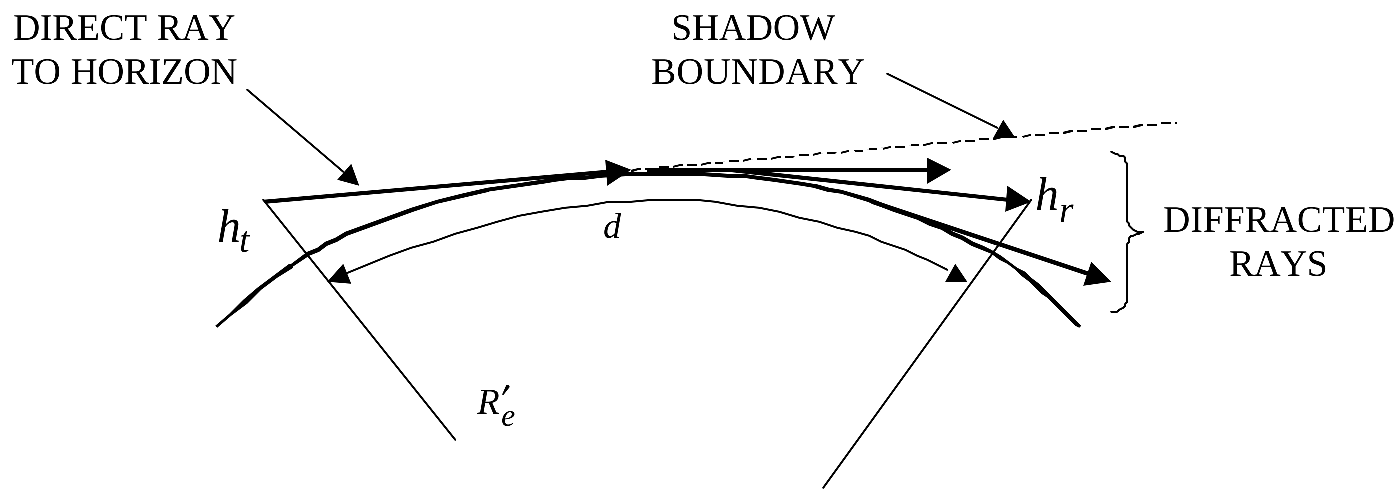
When can we use the flat Earth formulas?

1. The phase of the flat earth formula should be within $0.1p$ of the spherical earth value:
$$\frac{2kh_1h_2}{d} - nzp \le 0.1p$$
2. The divergence factor should be close to unity: typically $D \ge 0.9$.
3. The grazing angle should be small: $y \to 0$ and $K(S,T) \ge 0.9$.

There is no simple answer, but usually the distance d is considerably less than the radio horizon , d_{RH}, and frequently the two models disagree a distances as small as $d/d_{RH} \approx 1/3$. A comparison of the two models is shown.



Diffraction Region Formulas (1)



Approximate formulas for the diffraction region when f > 100 MHz:

$$F = V_1(X)U_1(Z_1)U_1(Z_2)$$

where U_1 is available from tables or curves, $Z_i = h_i / H \quad (i = 1,2)$, $X = d / L$, and

$$V_1(X) = 2\sqrt{pX}\,e^{-2.02X}, \; L = 2\left(\frac{(R'_e)^2}{4k}\right)^{\frac{1}{3}} = 28.41\,l^{1/3}(\mathrm{km}), \; H = \left(\frac{R'_e}{2k^2}\right)^{\frac{1}{3}} = 47.55\,l^{2/3}(\mathrm{m})$$

Diffraction Region Formulas (2)

A plot of $U_1(Z)$

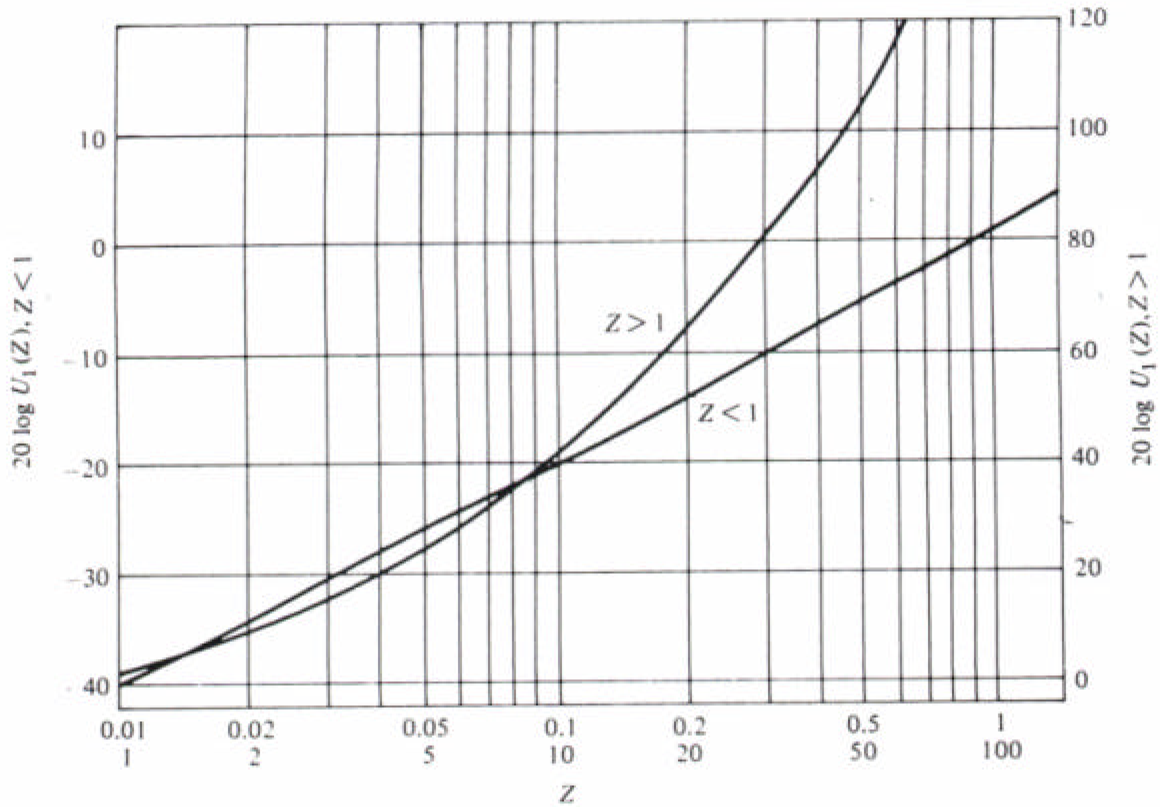


Fig. 6.29 in R. E. Collin, *Antennas and Radiowave Propagation*, McGraw-Hill, 1985 (axis labels corrected)

Surface Waves (1)

The behavior of electromagnetic waves at the Earth's surface is a complex function of frequency, polarization, ground characteristics, curvature of the surface, atmospheric conditions, and antenna height.

At ground level the direct and reflected waves nearly cancel. They do not completely cancel because the Earth is not a perfect conductor, and hence the antenna image current is not as intense as the source current.

At low frequencies (between about 1 kHz and 3 MHz) surface waves can be excited. Surface waves are solutions to the wave equation and the boundary conditions and, as the name implies, are guided along the boundary between two dissimilar materials. This fact is often used in designing surface waveguides that use a dielectric layer over a conducting ground plane.

Collectively the space wave (direct and Earth reflected) and surface wave are called the ground wave. At low frequencies, for vertical antennas near the ground, (e.g., AM broadcast), propagation is primarily via the surface wave. The rigorous solution for a dipole radiating over a lossy flat Earth was done by Sommerfeld, and later evaluated in a more convenient form by Norton[1].

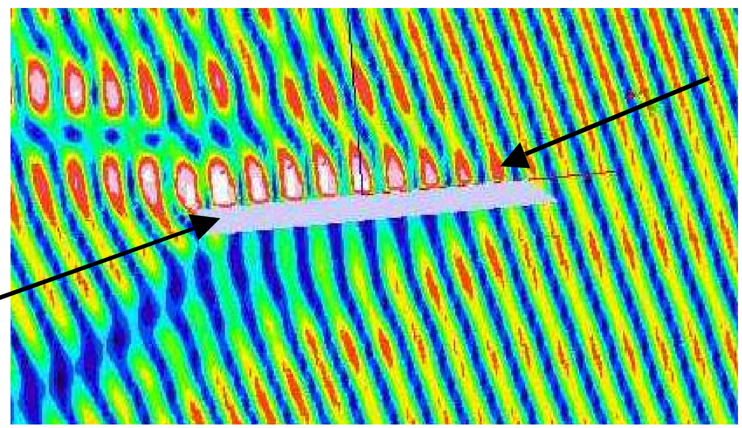
---

[1] K. Norton, "The Propagation of Radio Waves over the Surface of the Earth and in the Upper Atmosphere," *Proc. IRE*, v. 24, 1936

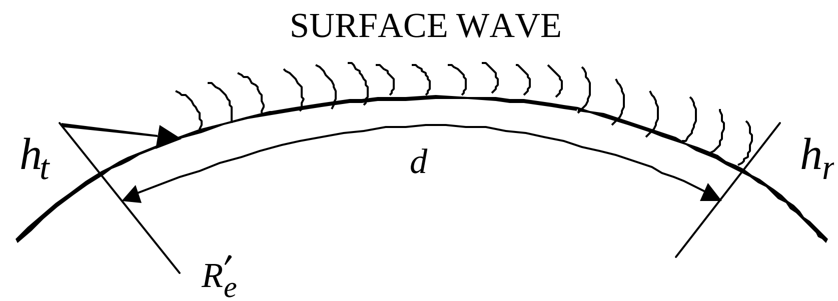
Surface Wave Illustration

Shown here is an example of a surface wave traveling along a coated conducting plate

- $5l$ plate
- 15 degree grazing angle
- TM (vertical) polarization
- the total field is plotted (incident plus scattered)
- surface waves will follow curved surfaces if the radius of curvature >> l



Surface Waves (2)



Simple formulas were derived by Norton. The power density at the receiver is the free space value times an attenuation factor

$$P_r = P_{\text{dir}} \left|2 A_s\right|^2$$

where the factor of 2 is by convention. The surface wave attenuation factor is

$$A_s = 1 - j\sqrt{p\ \Omega}\, e^{-\Omega} \operatorname{erfc}\left(j\sqrt{\ \Omega\ }\right)$$

where $\Omega = -jb\,\dfrac{d}{2e_r}\left(\sqrt{e_r} - 1\right)$ and erfc is the complementary error function. This calculation is for a surface wave along a flat interface which applies for $d \le 50/(f_{\text{MHz}})^{1/3}$ miles. Beyond this distance the received signal attenuates more quickly.

Surface Waves (2)

The parameter Ω is usually expressed as $\Omega = pe^{jb}$, where p is the numerical distance

$$p = \frac{kd}{2\sqrt{e_r^2 + (s\ /we_o)^2}} \quad \text{and} \quad b = \tan^{-1}\left(\frac{e_r e_o w}{s}\right)$$

A convenient formula is $s\ /we_o = \dfrac{1.8\times 10^4 s}{f_{\mathrm{MHz}}}$. The attenuation factor for the ground wave is approximately

$$|A_s| = \frac{2 + 0.3p}{2 + p + 0.6p^2} - \sqrt{p/2}\, e^{-0.6p} \sin b \qquad (b \le 90^\circ)$$

Example: A CB link operates at 27 MHz with low gain antennas near the ground. The following parameters hold: $P_t = 5$ W; $G_t = G_r = 1$; ground parameters, $e_r = 12$ and $s = 5\times 10^{-3}$ S/m. Find the received power at the maximum flat Earth range.

The maximum flat Earth distance is: $d_{\max} = 50/(27)^{1/3} = 16.5$ miles

Surface Waves (3)

Check b to see if the formula for p applies

$$b = \tan^{-1}\left(\frac{(12)(8.85\times10^{-12})(2p)(27\times10^{6})}{5\times10^{-3}}\right) = 74.5^{\circ} \quad \text{(yes)}$$

$$p = \frac{pd/l}{\sqrt{12^2 + (90/27)^2}} = 0.25\,d/l = 0.0225\left(\frac{16.5}{0.62}\right)(1000) \approx 601$$

From the above expression we also get $d/l = 4p$. The surface wave attenuation coefficient is

$$|A_s| = \frac{2+0.3p}{2+p+0.6p^2} - \sqrt{p/2}\,e^{-0.6p}\sin b \approx 8.33\times10^{-4}$$

The received power for the ground wave is

$$P_r = P_{\text{dir}}|2A_s|^2 = \frac{P_t G_t A_{er}}{4p d^2}|2A_s|^2 = \frac{P_t(1)\left(l^2/4p\right)}{4p d^2}|2A_s|^2$$

$$= \frac{(5)(8.33\times10^{-4})^2}{(4p)(4)^2(601)^2} = 1.52\times10^{-14}\text{ W}$$

Surface Waves (4)

FLAT EARTH

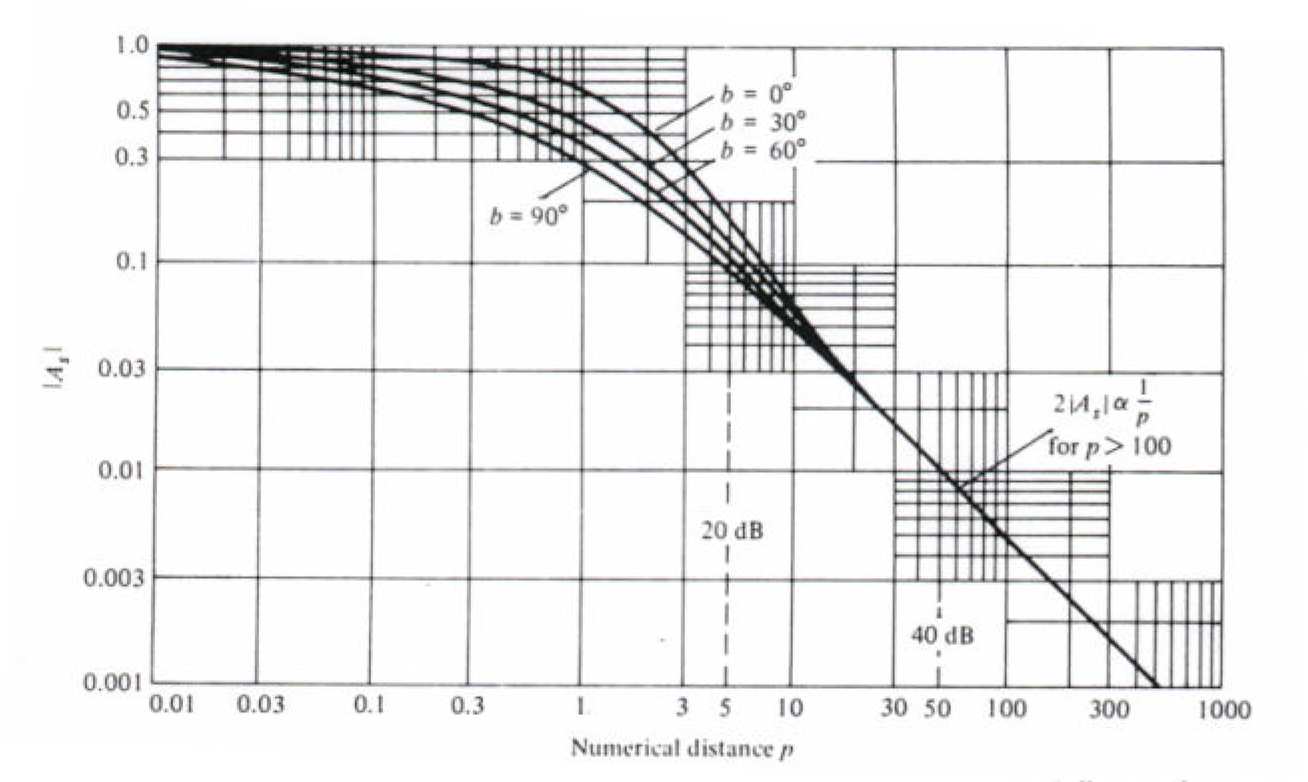


Fig. 6.35 in R. E. Collin, *Antennas and Radiowave Propagation*, McGraw-Hill, 1985

Surface Waves (5)

SPHERICAL EARTH ($\epsilon_r = 15$ and $\sigma = 10^{-2}$ S/m)

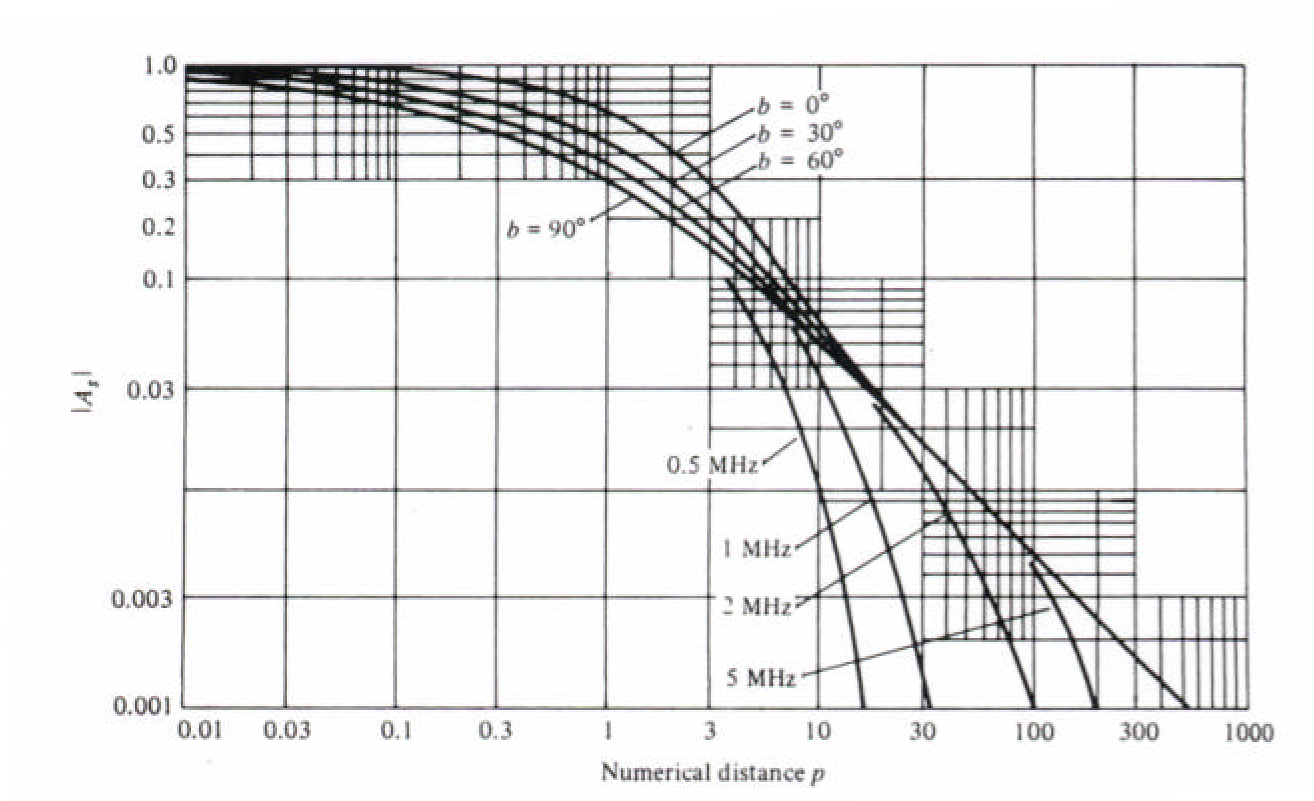
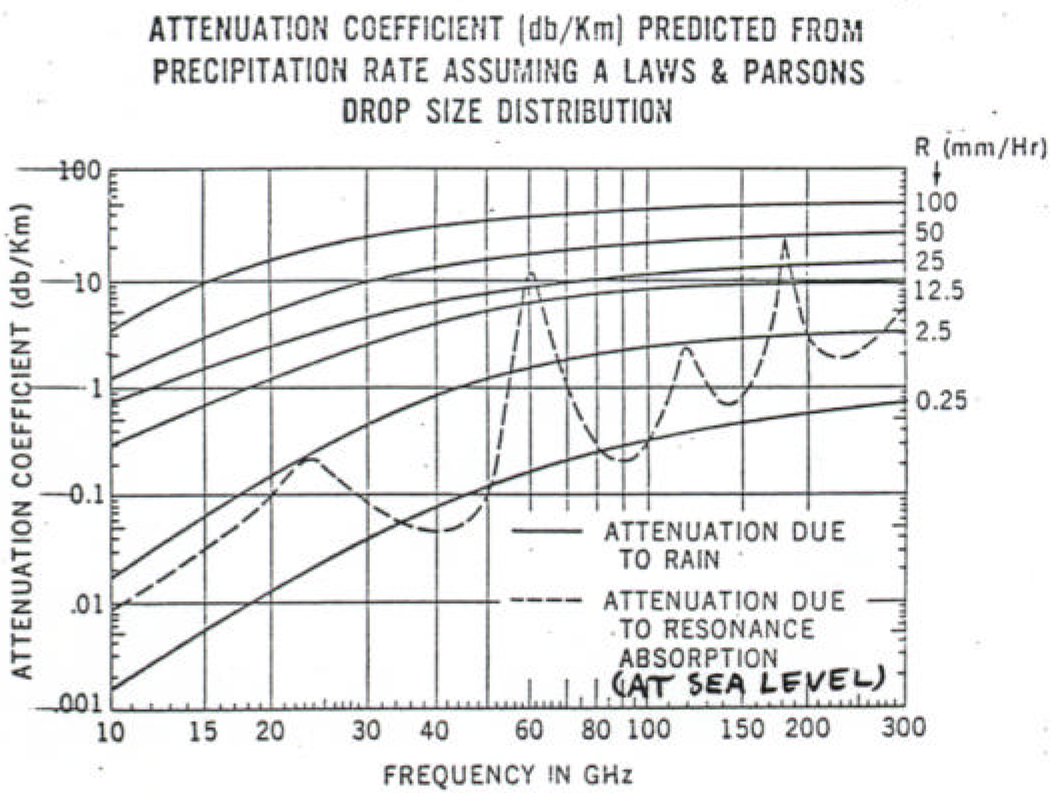


Fig. 6.36 in R. E. Collin, *Antennas and Radiowave Propagation*, McGraw-Hill, 1985

Attenuation Due to Rain and Gases (1)

Sources of signal attenuation in the atmosphere include rain, fog, water vapor and other gases. Most loss is due to absorption of energy by the molecules in the atmosphere. Dust, snow, and rain can also cause a loss in signal by scattering energy out of the beam.



Attenuation Due to Rain and Gases (2)

There is no complete, comprehensive macroscopic theoretical model to predict loss. A wide range of empirical formulas exist based on measured data. A typical model:

$A = aR^b$, attenuation in dB/km

R is the rain rate in mm/hr

$$a = G_a \, f_{\text{GHz}}{}^{E_a}$$

$$b = G_b \, f_{\text{GHz}}{}^{E_b}$$

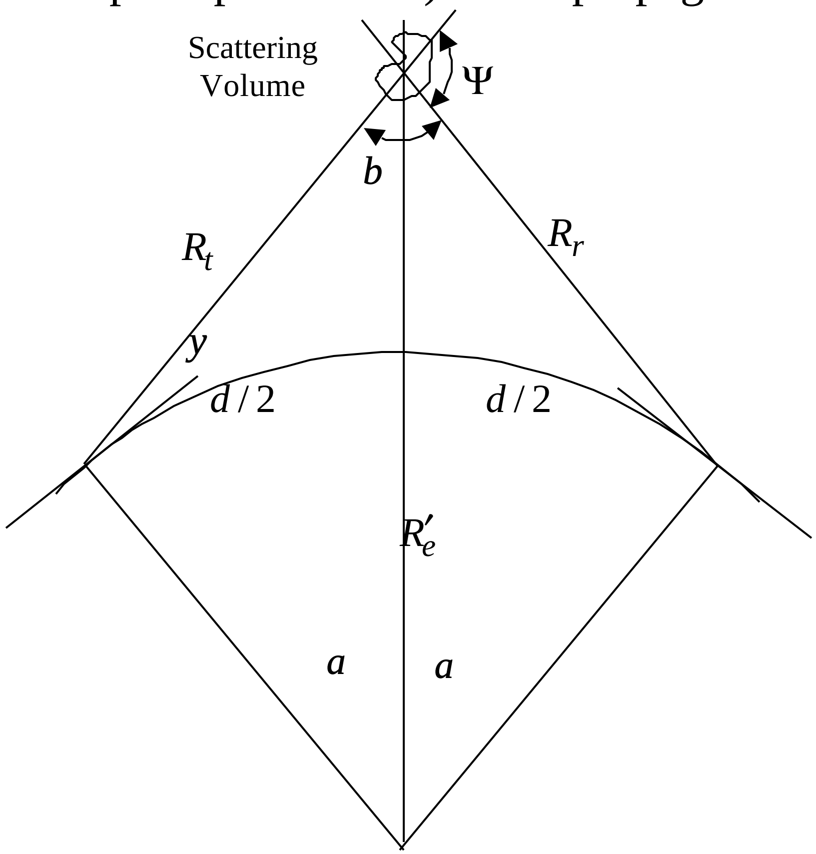
where the constants are determined from the following table:

$$\begin{aligned}
G_a &= 6.39\times10^{-5} & E_a &= 2.03 & f_{\text{GHz}} &< 2.9 \\
&= 4.21\times10^{-5} & &= 2.42 & 2.9 \le f_{\text{GHz}} &< 54 \\
&= 4.09\times10^{-2} & &= 0.699 & 54 \le f_{\text{GHz}} &< 180
\end{aligned}$$

$$\begin{aligned}
G_b &= 0.851 & E_b &= 0.158 & f_{\text{GHz}} &< 8.5 \\
&= 1.41 & &= -0.0779 & 8.5 \le f_{\text{GHz}} &< 25 \\
&= 2.63 & &= -0.272 & 25 \le f_{\text{GHz}} &< 164
\end{aligned}$$

Tropospheric Scatter Propagation (1)

Tropospheric scatter propagation (troposcatter) exploits fluctuations in the dielectric constant of the atmosphere due to variations in temperature and pressure (typically only a few parts per million). This propagation mechanism was one of the few methods available for communicating long distances over the horizon (along with ionospheric hops) before the advent of satellite communications. A few troposcatter systems are still in use today.



A common geometry is shown in the figure. High gain narrow beam antennas are used. For maximum range the scattering volume is midway between the transmitter and receiver. The antenna pointing angle above the horizon y is small so that $R_t = R_r \approx d/2$. b is called the bistatic angle. From geometry

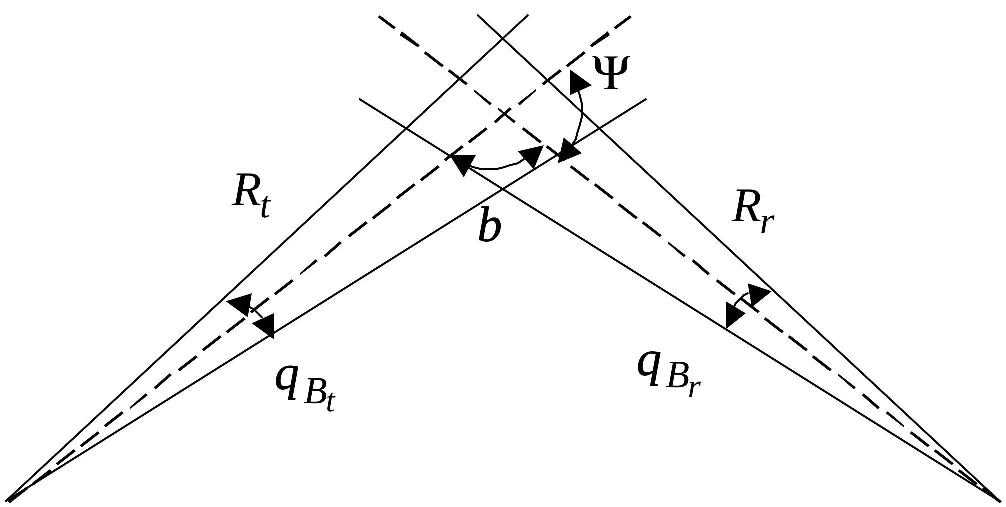
$$\Psi + b = p \quad \text{and} \quad a = d/(2R_e')$$

Tropospheric Scatter Propagation (2)

The received power is given by the bistatic radar range equation

$$P_r = \frac{P_t G_t G_r s_V l^2}{(4p)^3 R_t^2 R_r^2} = \frac{P_t G^2 s_V}{4p\, k^2 (d/2)^4}$$

where $k = 2p / l$ and s_V is the scattering cross section of the volume of the intersection of the beams. The scattering volume is the cross sectional area of a parallelogram (shown in the elevation cut below), times the thickness in azimuth. If all of the beamwidths are the same, then the volume between the beamwidths is



$$V_c = \frac{(Rq_B)^2}{\sin b} Rq_B \approx \left(\frac{dq_B}{2}\right)^3 \frac{1}{\sin b}$$

The maximum scattering occurs in the forward direction, $\Psi = 0$. In other directions is falls off as $\cos^2 \Psi$.

Tropospheric Scatter Propagation (3)

The scattering cross section (m^2) can be expressed as

$$\sigma_V = \frac{\pi^5}{\lambda^4} V_c S \cos^2 \Psi$$

where S is the spatial scattering spectrum. It is related to the correlation function of the time varying index of refraction. The λ^{-4} dependence is characteristic of Rayleigh scattering, which occurs when the scattering particles are much less that a wavelength. An approximation for S is

$$S = 32\pi^3 C_n^2 \left(3.3\times10^{-2}\right)\left(2k\sin(\Psi/2)\right)^{-11/3} \quad \mathrm{m}^3$$

C_n is called the structure constant for the index of refraction fluctuations. At a height of 1.5 km it takes on values $2\times10^{-8} \le C_n \le 5\times10^{-7}\ \mathrm{m}^{-1/3}$. Two further approximations are also valid: $2\sin(\Psi/2) \approx \Psi \approx d/R_e'$ and $\sin\beta \approx \beta \approx d/R_e'$. Therefore the radar range equation becomes

$$P_r = \frac{P_t G^2}{4\pi k^2 (d/2)^4} \frac{\pi^5}{\lambda^4} 32\pi^3 C_n^2 \left(3.3\times10^{-2}\right)\left(kd/R_e'\right)^{-11/3} \left(\frac{d\theta_B}{2}\right)^3 \frac{\cos^2\Psi}{d/R_e'}$$

$$P_r \approx 2.23\times10^{32} P_t G^2 C_n^2 k^{-5/3} d^{-17/3} \theta_B^3 \cos^2 \beta$$

Tropospheric Scatter Propagation (4)

Example: A tropospheric communication link operates at 10 cm with a transmit power of 30 dB, parabolic reflector antenna gains of 50 dB (50% efficiency), and a transmit/receive distance of 400 km. Find the received power for $C_n = 10^{-8}\ \mathrm{m}^{-1/3}$.

The antenna diameter is related to the gain

$$G = 4p\left(pD^2/4\right)e/l^2 \quad \rightarrow \quad D = \left[10^5(0.1)^2/(p^2\,0.5)\right]^{1/2} = 14.2\ \mathrm{m}$$

The HPBW can be estimated from the product of the beamwidths

$$G = 4pe/\Omega_A \approx 4pe/q_B^2 \quad \rightarrow \quad q_B = \left[4pe/10^5\right]^{1/2} = 0.008\ \mathrm{rad}$$

The received power is

$$P_r \approx 2.23\times 10^{32}10^3 10^{10}\left(10^{-8}\right)^2\left(\frac{2p}{0.1}\right)^{-5/3}\left(400\times 10^3\right)^{-17/3}0.008^3 = 2\times 10^{-12}\ \mathrm{W}$$

Note that for a LOS free space propagation link $\dfrac{P_t G^2 l^2}{(4p)^2 d^2} = 6.3\times 10^{-2}$ W. The troposcatter link has an additional 105 dB of loss.

Tropospheric Scatter Propagation (5)

In practice, more simple models of troposcatter links are used. Empirical or measured loss factors are applied to the Friis equation. For example,

$$P_r = P_t G_t G_r \left(\frac{l}{4pR} \right)^2 L_c L_s L_f \quad \rightarrow \quad P_{r_{\mathrm{dB}}} = P_{t_{\mathrm{dB}}} + G_{t_{\mathrm{dB}}} + G_{r_{\mathrm{dB}}} - L_{p_{\mathrm{dB}}} - L_{c_{\mathrm{dB}}} - L_{s_{\mathrm{dB}}} - L_{f_{\mathrm{dB}}}$$

where: $L_p = \left(\frac{4pR}{l} \right)^2$ = path loss

L_s = scatter loss

L_c = antenna to medium coupling loss

L_f = feeder loss (transmission line or pluming loss)

The losses are defined such that they are positive quantities in dB. Empirical formulas for the coupling and scattering losses are:

$$L_{s_{\mathrm{dB}}} = 83 + 0.57\Psi_{\mathrm{mrad}} + 10\log f_{\mathrm{MHz}} - 0.2N$$
$$L_{c_{\mathrm{dB}}} = 0.07\exp\left[0.055\left(G_{t_{\mathrm{dB}}} + G_{r_{\mathrm{dB}}}\right)\right]$$

N is the <u>surface refractivity</u>, a quantity that will be discussed in more detail in the lectures on ducting and nonstandard refraction.

Tropospheric Scatter Propagation (6)

Example: Rework the last example using the simplified model (50 dB gains, 30 dB power, 3 GHz, and d = 400 km). Assume a refractivity of 320.

The angle Ψ must be determined in milliradians. From geometry $a = d/(2R'_e)$, $b = p - d/R'_e$ and $\Psi = p - b = d/R'_e = 0.0468$ rad = 46.8 mrad

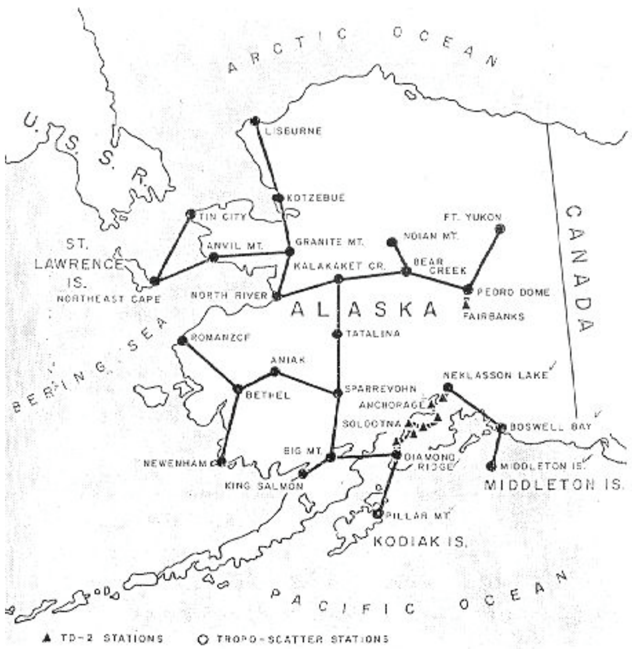
$$L_{s_{\mathrm{dB}}} = 83 + 0.57(46.8) + 10\log 3000 - 0.2(320) = 80.5\,\mathrm{dB}$$
$$L_{c_{\mathrm{dB}}} = 0.07\exp[0.055(100)] = 17.1\,\mathrm{dB}$$
$$L_{p_{\mathrm{dB}}} = -20\log\left[0.1/(4p\,400\times10^3)\right] = 154\,\mathrm{dB}$$

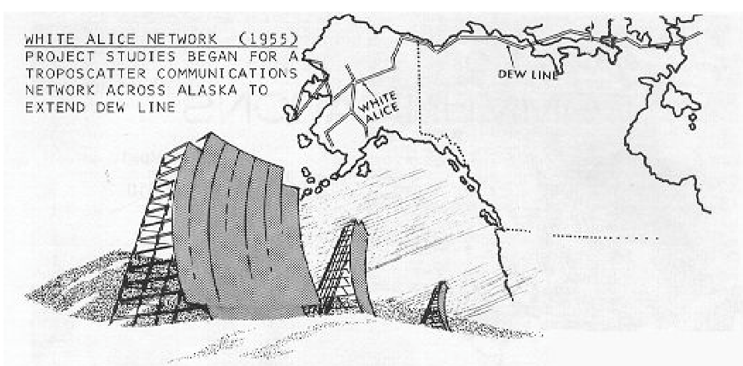
The received power is

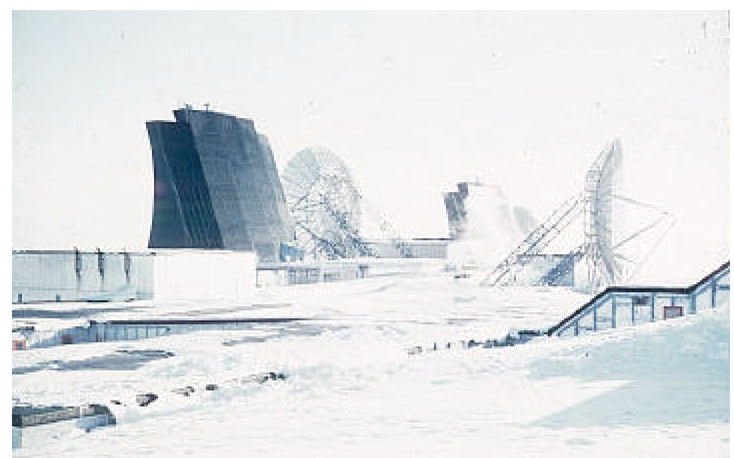
$$P_{r_{\mathrm{dB}}} = 30 + 50 + 50 - 154 - 17.1 - 80.5 = -121.6\,\mathrm{dB} = 6.9\times10^{-13}\ \mathrm{W}$$

This is a difference of 4.6 dB from the previous result. For this application it can be considered good agreement. Variations in the atmosphere cause the signal strength to vary by perhaps tens of dB over relatively short periods of time.

White Alice Troposcatter Link

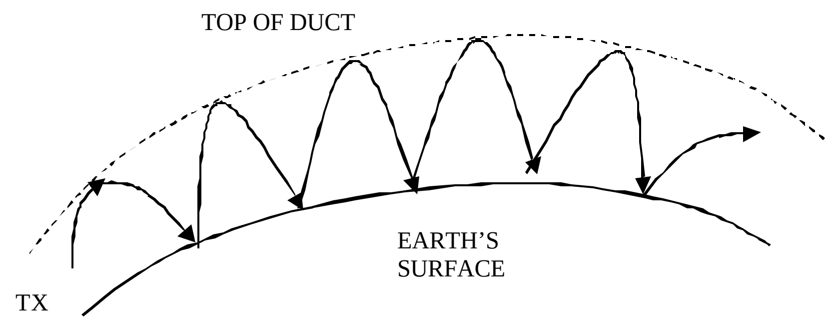






Ducts and Nonstandard Refraction (1)

Nonstandard propagation takes place when the index of refraction deviates significantly from the linear decrease with height assumed for the standard atmosphere. Relatively large changes over short heights (10s to 100s of meters) result in ducts. Rays launched at the proper angle and frequency will be confined to the duct. Similar to a waveguide, waves in a duct do not suffer a $1/R^2$ isotropic spreading loss.



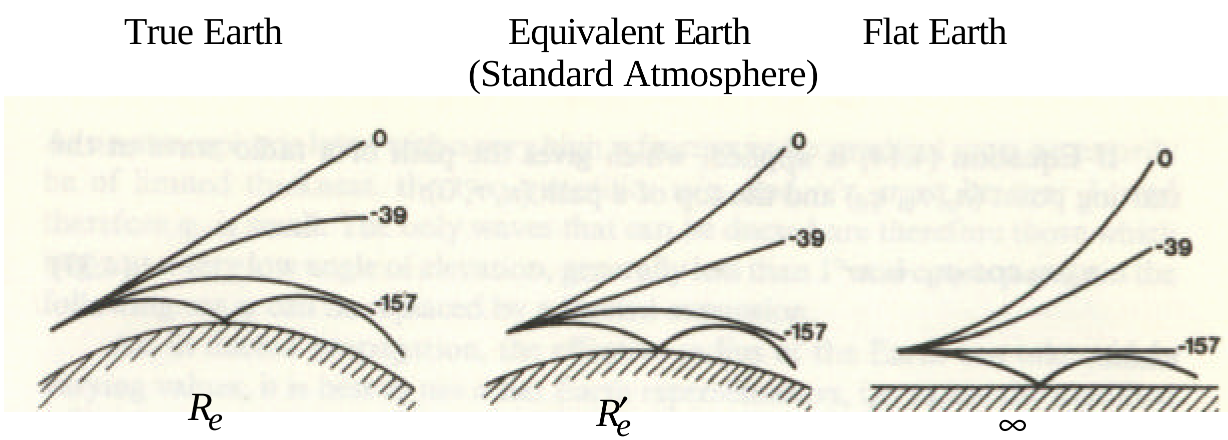
- The formation of ducts is due primarily to water vapor, and therefore they tend to occur over bodies of water (but not land-locked bodies of water)
- They can occur at the surface or up to 5000 ft (elevated ducts)
- Thickness ranges from a meter to several hundred meters
- The trade wind belts have a more or less permanent duct of about 1 to 5 m thickness
- Efficient propagation occurs for UHF frequencies and above if both the transmitter and receiver are located in the duct
- If the transmitter and receiver are not in the duct, significant loss can occur before coupling into the duct

Ducts and Nonstandard Refraction (2)

Because variations in the index of refraction are so small, a quantity called the refractivity is used

$$N(h) = [n(h) - 1]10^6 \qquad n(h) = \sqrt{e_r(h)}$$

In the normal (standard) atmosphere the gradient of the vertical refractive index is linear with height, $dN/dh \approx -39$ N units/km. If $dN/dh < -157$ then rays will return to the surface. Rays in the three Earth models are shown below.



From *Radiowave Propagation*, Lucien Boithias, McGraw-Hill

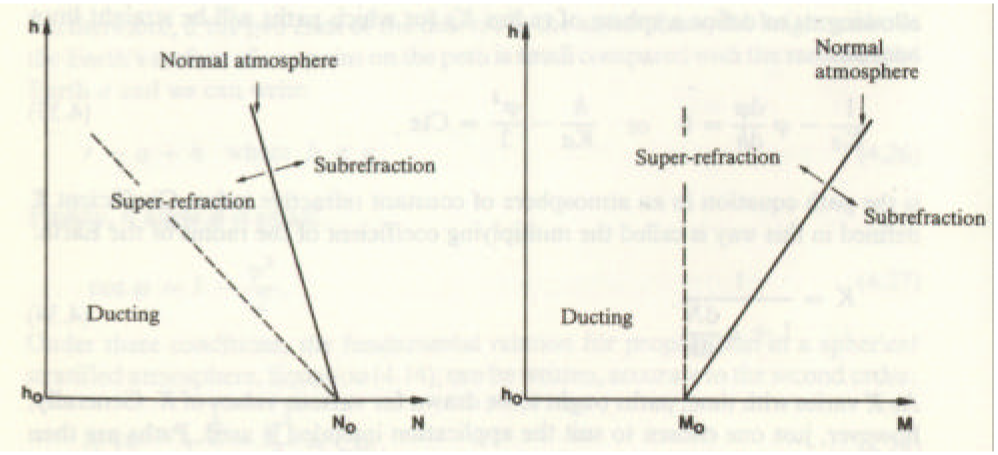
Ducts and Nonstandard Refraction (3)

Another quantity used to solve ducting problems is the modified refractivity

$$M(h) = N(h) + 10^6 (h / R_e')$$

In terms of *M*, the condition for ducting is $dM/dh = dN/dh + 157$. Other values of dN/dh (or dM/dh) lead to several types of refraction as summarized in the following figure and table. They are:

1. Super refraction: The index of refraction decrease is more rapid than normal and the ray curves downward at a greater rate
2. Substandard refraction (subrefraction): The index of refraction decreases less rapidly than normal and there is less downward curvature than normal



From *Radiowave Propagation*, Lucien Boithias, McGraw-Hill

Ducts and Nonstandard Refraction (4)

Summary of refractivity and ducting conditions

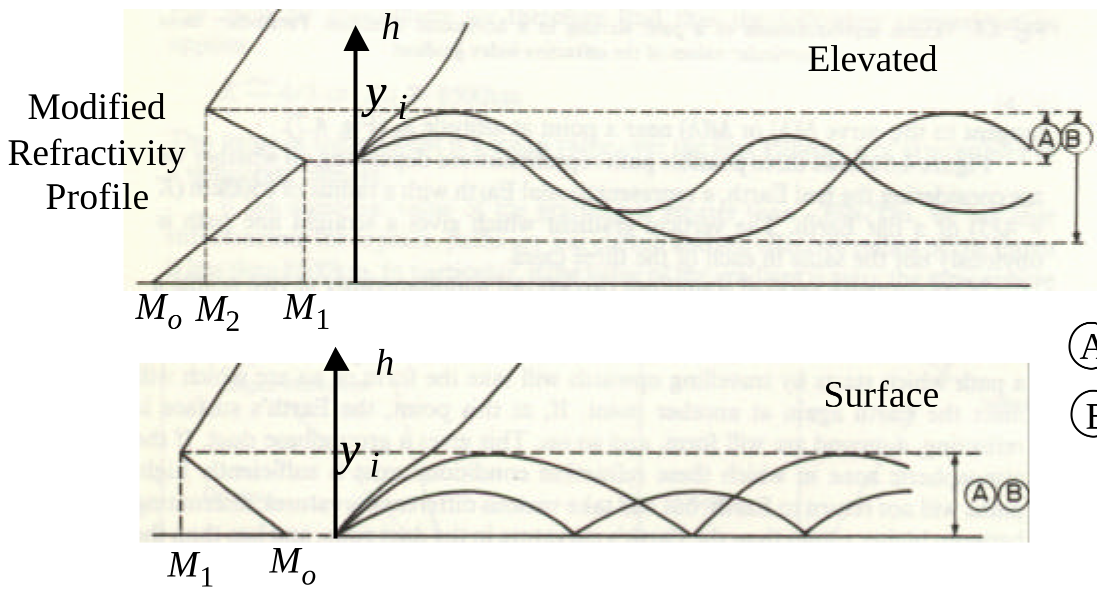
dN/dh	Ray Curvature	k	Atmospheric Refraction	Virtual Earth	Horizontally Launched Ray
> 0	up	< 1	below normal	more convex	moves away from Earth
0	none	1		actual	
$0 > \dfrac{dN}{dh} > -39$	down	> 1		less convex	
-39		4/3	normal		
$-39 > \dfrac{dN}{dh} > -157$		> 4/3	above normal		
-157				plane	parallel to Earth
< -157			super-refraction	concave	draws closer to Earth

Ducts and Nonstandard Refraction (5)

Using a stratified model for the duct, the spherical form of Snell's Law must hold at the top and bottom. Let (ψ_i, R_i, n_i) be the launch angle, distance to the Earth center, and index of refraction at the bottom, and similarly $(90^\circ, R, n)$ at the top. Then

$$n_i R_i \sin \psi_i = nR \sin 90^\circ \quad \rightarrow \quad \sin \psi_i = \frac{nR}{n_i R_i} \approx 1$$

Thus the incidence angle must also be very close to 90° (generally $< 89^\circ$). Some examples of paths in a duct are shown below.

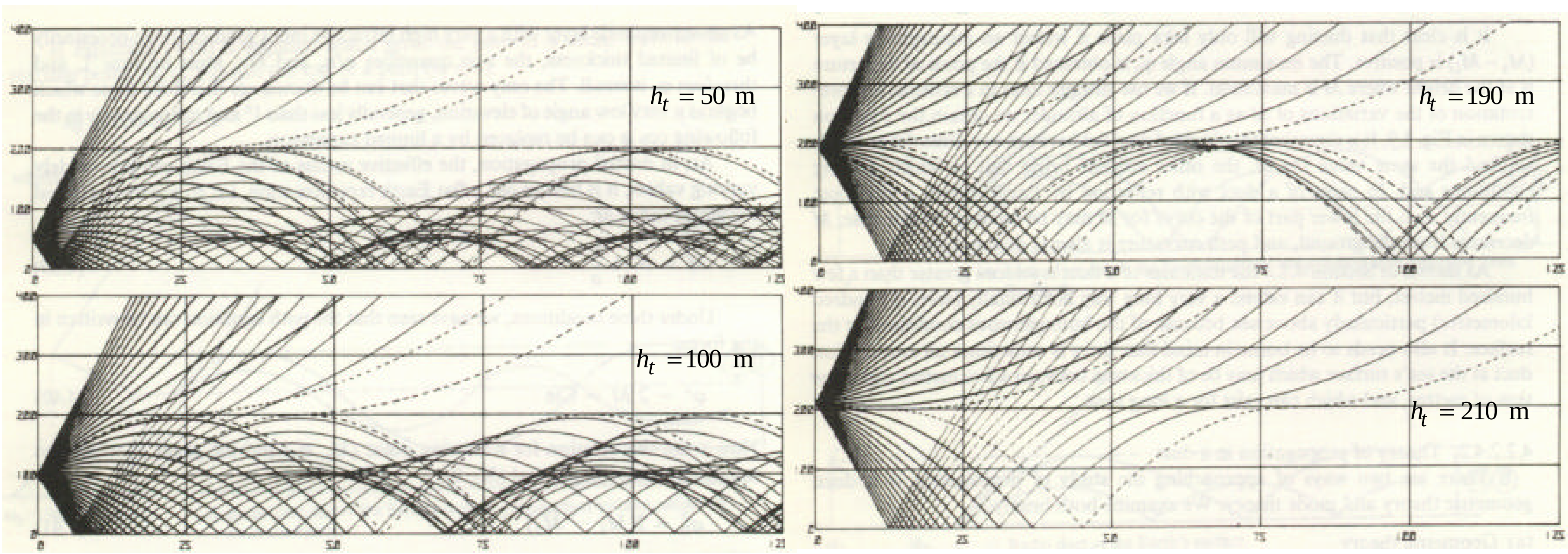


Ⓐ Ducting Layer

Ⓑ Tropospheric Radio Duct

Ducts and Nonstandard Refraction (6)

Some examples of ray trajectories in ducts (gradient of the duct –500 N/km, thickness 200m) for various transmitter heights.



From *Radiowave Propagation*, Lucien Boithias, McGraw-Hill

Ducts and Nonstandard Refraction (7)

Approximate cutoff frequency of the duct is

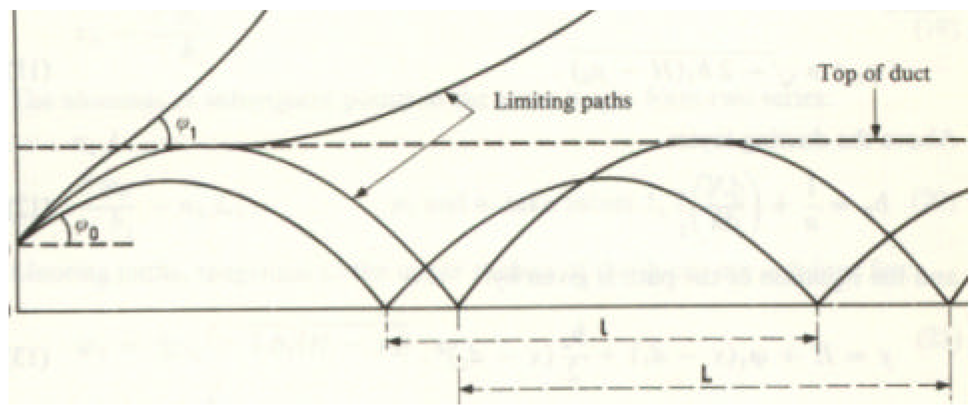
$$l_{\max} = 2.5(\Delta h)\sqrt{\Delta n} \quad \rightarrow \quad f_{\min} = c \,/\, l_{\max}$$

$\Delta h =$ depth of the duct, and $\Delta n =$ variation of the index inside of the duct. Note that the cutoff is not abrupt, as in a waveguide with conducting walls. As the frequency falls below cut-off the duct lets more and more energy escape. Refractive index variations near the top of the duct make the cutoff frequency even less definite.

The lengths of the arcs of a ducted path are approximately

$$L = 2\sqrt{\frac{2000\,(\Delta h)}{-(dN/dh + 157)}} \text{ km}$$

where Δh is in meters and dN/dh is in N/km.



Ducts and Nonstandard Refraction (8)

Example: A duct near the ground with a thickness of 30 m and $\Delta n = 4 \times 10^{-6}$ (4 N units), a typical case at middle latitudes.

$$l_{\max} = 2.5(30)\sqrt{4 \times 10^{-6}} = 0.15\,\mathrm{m} \quad \rightarrow \quad f_{\min} = \frac{c}{l_{\max}} = 2000\ \mathrm{MHz}$$

Example: Find the values of L for duct heights of 100, 200 and 300 m and dN/dh = –160, –200 and –500 N/km.

Using the formula on the previous page gives the following results:

dN/dh	$\Delta h = 100$	$\Delta h = 200$	$\Delta h = 300$
–160	516	730	894
–200	136	192	236
–500	48	68	84

Ducts and Nonstandard Refraction (9)

From the definition of refractivity: $N(h) = [n(h) - 1]10^6$ so

$$\frac{dN}{dh} = \frac{dn}{dh}10^6 \text{ N/m} = \frac{dn}{dh}10^9 \text{ N/km}$$

Note that from p. 21:

$$k = \left[1 + \frac{R_e}{\sqrt{e_o}}\frac{d\sqrt{e}}{dh}\right]^{-1} = \left[1 + R_e\frac{dn}{dh}\right]^{-1} = \left[1 + 10^{-9} R_e \frac{dN}{dh}\right]^{-1}$$

Example: At what range could a target at a height of 1km be detected by a radar at sea level if $\frac{dN}{dh} = -20$ N/km?

$$k = \left[1 + 10^{-9} R_e \frac{dN}{dh}\right]^{-1} = \left[1 + 10^{-9}(6375 \times 10^3)(-20)\right]^{-1} = 1.146$$

$$d = \sqrt{2kR_e h} = \sqrt{2(1.146)(6375)(1)} = 120.9 \text{ km}$$

AREPS Simulation of a Radar in a Duct

AREPS (Advanced Refractive Effects Prediction) software available from SPAWAR PMW-155 (San Diego)